7.25 A linear time-invariant discrete-time system has transfer function

$$H(z) = \frac{z}{(z-0.5)^2(z^2+0.25)}$$

(a) Find the unit-pulse response $h[n]$ for all $n \geq 0$.
(b) Simulate the unit-pulse response by using MATLAB, and compare this result with the result for $h[n]$ obtained analytically, in part (a).

- For b), plot the answer from a) using MATLAB (i.e., partial fractions solution) for $0 \leq n \leq 10$ and on another stem plot show the answer found using long division (can use dimpulse.m function). Place the partial fractions solution plot on the top and the long division answer plot on the bottom of the same page. Attach m-file(s). Are the plots the same? Is the system stable or not? Why?

a) Find roots of denominator + divide both sides by z

$$\frac{H(z)}{z} = \frac{1}{z^4 - z^3 + 0.5z^2 - 0.25z + 0.0625} = \frac{1}{(z-0.5)^2(z-j0.5)(z+j0.5)}$$

$$= \frac{C_1}{z-0.5} + \frac{C_2}{(z-0.5)^2} + \frac{C_3}{z-j0.5} + \frac{C_4}{z+j0.5}$$

$$C_4 = \left[(z+j0.5)\frac{H(z)}{z}\right]\Bigg|_{z=-j0.5} = \frac{1}{(-j0.5-0.5)^2(-j0.5-j0.5)} = \underline{2}$$

$$C_3 = C_4^* = \underline{2}$$

$$C_2 = \left[(z-0.5)^2\frac{H(z)}{z}\right]\Bigg|_{z=0.5} = \frac{1}{(0.5-j0.5)(0.5+j0.5)} = \underline{2}$$

$$C_1 = \left[\frac{d}{dz}\left\{(z-0.5)^2\frac{H(z)}{z}\right\}\right]\Bigg|_{z=0.5} = \left[\frac{d}{dz}\left(\frac{1}{z^2+0.25}\right)\right]\Bigg|_{z=0.5}$$

$$= \frac{(-1)2z}{(z^2+0.25)^2}\Bigg|_{z=0.5} = \frac{(-1)(1)}{(0.25+0.25)^2} = \underline{-4}$$

a) cont.

$$\frac{H(z)}{z} = \frac{-4}{z-0.5} + \frac{2}{(z-0.5)^2} + \frac{2}{z-j0.5} + \frac{2}{z+j0.5}$$

$$H(z) = -4\frac{z}{z-0.5} + 2\frac{z}{(z-0.5)^2} + \frac{2z}{z-j0.5} + \frac{2z}{z+j0.5}$$

Use transform pairs, $a^n u[n] \longleftrightarrow \frac{z}{z-a}$ (Table 7.3)

$$c_2 n p_1^{n-1} u[n] \longleftrightarrow \frac{c_2 z}{(z-p_1)^2} \quad (7.73)$$

and linearity to get:

$$h[n] = -4(0.5)^n u[n] + \underset{\substack{\uparrow \\ \text{factor out } (0.5)^{-1}}}{2n(0.5)^{n-1}} u[n] + \underbrace{2(j0.5)^n u[n] + 2(-j0.5)^n u[n]}_{\substack{\text{Combine (7.68)} \\ 2|2||j0.5|^n \cos(\frac{\pi}{2}n + 0)}}$$

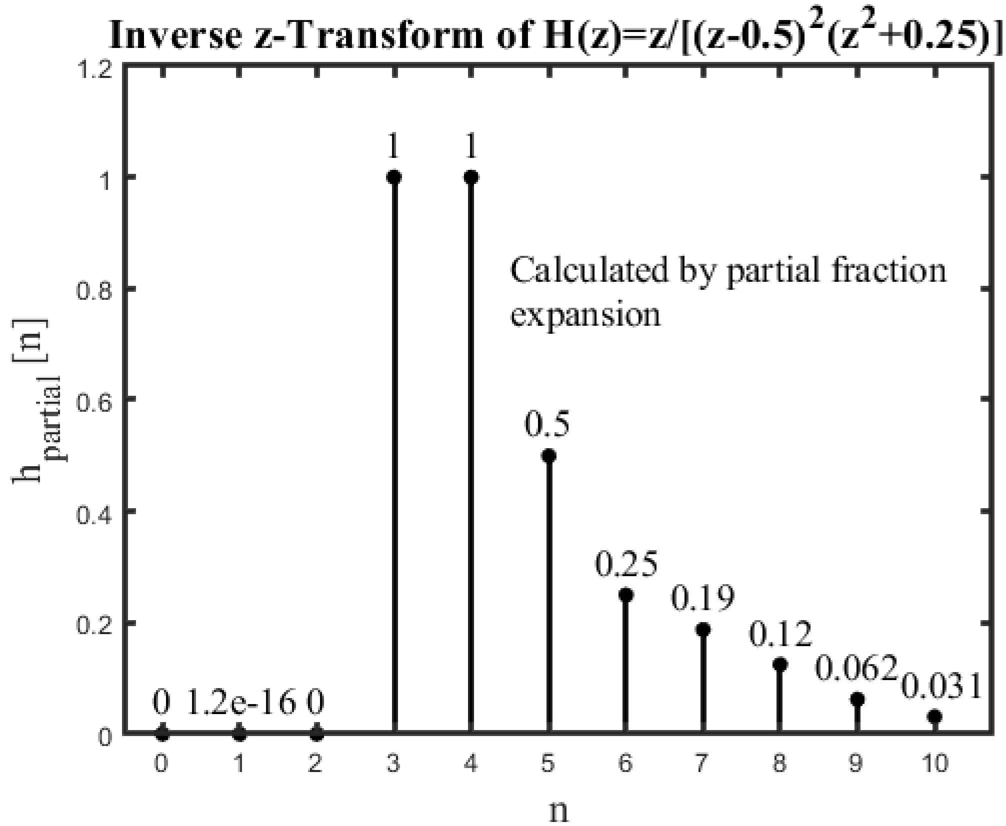
$$\underline{\underline{h[n] = -4(0.5)^n u[n] + 4n(0.5)^n u[n] + 4(0.5)^n \cos(\tfrac{\pi}{2}n) u[n]}}$$

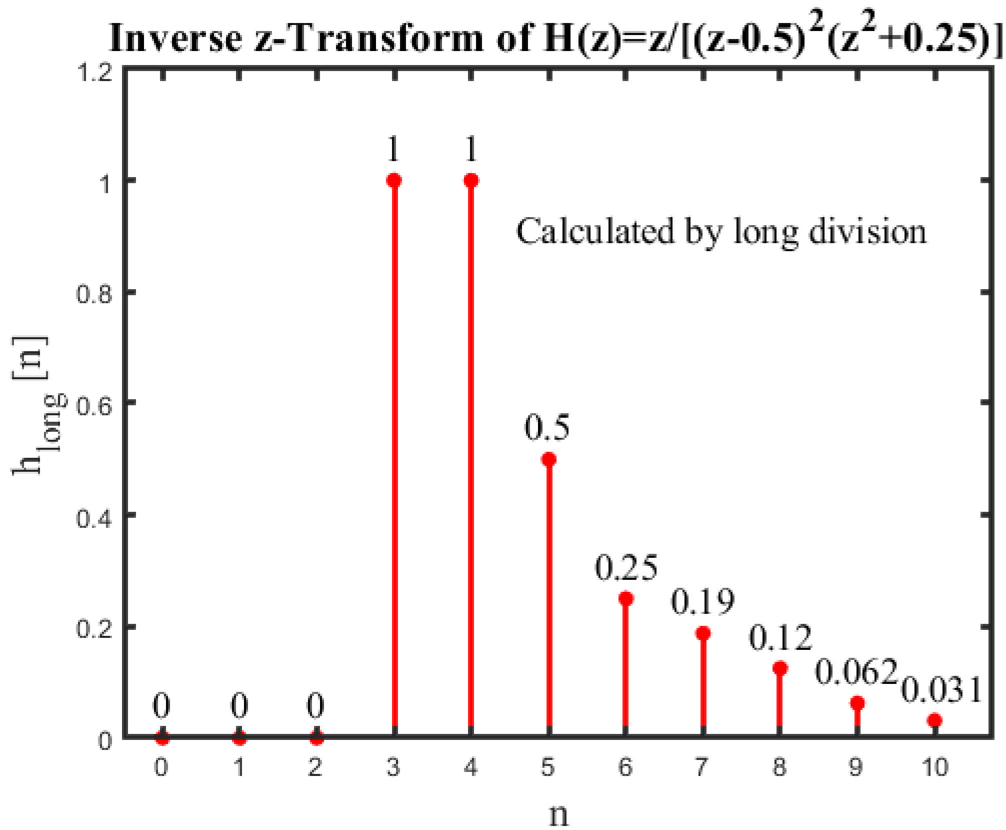
→ System is stable $h[n\to\infty] \to 0$

all pole w/in $|z|=1$ circle

→ Both plots identical

b)


Inverse z-Transform of H(z)=z/[(z-0.5)^2(z^2+0.25)]
Calculated by partial fraction expansion
h_partial [n]
n
0 1.2e-16 0
1 1
0.5
0.25
0.19
0.12
0.062
0.031


Inverse z-Transform of H(z)=z/[(z-0.5)^2(z^2+0.25)]
Calculated by long division
h_long [n]
n
0 0 0
1 1
0.5
0.25
0.19
0.12
0.062
0.031

b) cont.

```
% p7_25.m
% Plot unit pulse response for
%   H(z)= z/[(z-0.5)^2 *(z^2+0.25)]
%        = z/[z^4-z^3+0.5z^2-0.25z+0.0625]
% using long division and by partial fractions solution
% h[n] = -4(0.5)^n u[n] + 4n(0.5)^n u[n]+ 4(0.5)^n cos(pi*n/2) u[n]
%
clc;clear;close all;
num = [1,0];     % Input coefficients of numerator polynomial
den = [1,-1,0.5,-0.25,0.0625]; % Input coeff. of denominator polynomial
% Calculate first 6 values of h[n] by long division
hlong = dimpulse(num,den,11);  n=0:1:10;
h_part = -4*(0.5).^n + 4*n.*(0.5).^n + 4*(0.5).^n.*cos(pi*n/2);
stem(n,h_part,'k.','linewidth',2,'markersize',20),
axis([-0.5 10.75 0 1.2]),
ylabel('h_{partial }[n]','fontsize',16,'fontname','times'),
xlabel('n','fontsize',16,'fontname','times'),
title('Inverse z-Transform of H(z)=z/[(z-0.5)^2(z^2+0.25)]',...
    'fontsize',16,'fontname','times'),
text(4.1,0.75,'Calculated by partial fraction expansion',...
    'fontsize',16,'fontname','times')
for m=1:length(h_part),
    text(n(m),h_part(m)+0.02,['',num2str(h_part(m),2)],...
        'horizontalalignment','center','verticalalignment','bottom')
end
figure, stem(n,hlong,'r.','linewidth',2,'markersize',20),
axis([-0.5 10.75 0 1.2]),
ylabel('h_{long }[n]','fontsize',16,'fontname','times'),
xlabel('n','fontsize',16,'fontname','times'),
title('Inverse z-Transform of H(z)=z/[(z-0.5)^2(z^2+0.25)]',...
    'fontsize',16,'fontname','times'),
text(4.1,0.75,'Calculated by long division',...
    'fontsize',16,'fontname','times')
for m=1:length(hlong),
    text(n(m),hlong(m)+0.02,['',num2str(hlong(m),2)],...
        'horizontalalignment','center','verticalalignment','bottom')
end
set(findobj('type','line'),'linewidth',1.5)
set(findobj('type','line'),'markersize',18)
set(findobj('type','axes'),'linewidth',2)
set(findobj('type','text'),'fontsize',14,'fontname','times')
```