7.24 A linear time-invariant discrete-time system has transfer function

$$H(z) = \frac{z^2 - z - 2}{z^2 + 1.5z - 1}$$

(a) Compute the unit-pulse response $h[n]$ for all $n \geq 0$.
(b) Compute the step response $y[n]$ for all $n \geq 0$.
(e) Verify the results of parts (a) to (d) by computer simulation.

- For e), plot the answers from a & b using MATLAB (i.e., partial fractions solutions) for $0 \leq n \leq 5$ and on another stem plot show the answer found using long division (can use dimpulse.m function). For each case, place the partial fractions solution plot on the top and the long division answer plot on the bottom of the same page. Label each stem. Attach m-files. Are the plots the same? Is the system stable or not? Why?

a) $H(z) = \frac{z^2 - z - 2}{(z-0.5)(z+2)}$ ⟵ found roots using TI-68

$$\frac{H(z)}{z} = \frac{c_0}{z} + \frac{c_1}{z-0.5} + \frac{c_2}{z+2}$$

$$c_0 = \left[z \frac{H(z)}{z}\right]\Big|_{z=0} = \frac{-2}{-1} = \underline{2}$$

$$c_1 = \left[(z-0.5) \frac{H(z)}{z}\right]\Big|_{z=0.5} = \frac{0.5^2 - 0.5 - 2}{(0.5)(0.5+2)} = \underline{-1.8}$$

$$c_2 = \left[(z+2) \frac{H(z)}{z}\right]\Big|_{z=-2} = \frac{(-2)^2 + 2 - 2}{(-2)(-2-0.5)} = \underline{0.8}$$

$$H(z) = 2 - 1.8 \frac{z}{z-0.5} + 0.8 \frac{z}{z+2}$$

use Table 7.3 transform pairs + linearity

$$a^n u[n] \Leftrightarrow \frac{z}{z-a}$$

$$\delta[n] \Leftrightarrow 1$$

a) cont.

$$h[n] = 2\delta[n] - 1.8(0.5)^n u[n] + 0.8(-2)^n u[n]$$

$\Rightarrow$ Not Stable $(-2)^n$ term $\rightarrow \infty$ as $n \rightarrow \infty$

b) Find step response $y[n]$ for all $n \geq 0$

By definition, $x[n] = u[n]$. By Table 7.3,

$$X(z) = \frac{z}{z-1}$$

for long division ↓

$$Y(z) = H(z)X(z)$$

$$= \frac{(z^2 - z - 2)z}{(z-0.5)(z+2)(z-1)} = \frac{z^3 - z^2 - 2z + 0}{z^3 + 0.5z^2 - 2.5z + 1}$$

$$\frac{Y(z)}{z} = \frac{c_1}{z-0.5} + \frac{c_2}{z-1} + \frac{c_3}{z+2}$$

$$c_1 = \left[(z-0.5)\frac{Y(z)}{z}\right]\Bigg|_{z=0.5} = \frac{0.5^2 - 0.5 - 2}{(0.5+2)(0.5-1)} = 1.8$$

$$c_2 = \left[(z-1)\frac{Y(z)}{z}\right]\Bigg|_{z=1} = \frac{1^2 - 1 - 2}{(1-0.5)(1+2)} = -1.\overline{3} = -4/3$$

$$c_3 = \left[(z+2)\frac{Y(z)}{z}\right]\Bigg|_{z=-2} = \frac{(-2)^2 + 2 - 2}{(-2-0.5)(-2-1)} = 0.5\overline{3} = 8/15$$

b) cont.

$$Y(z) = 1.8 \frac{z}{z-0.5} - \frac{4}{3}\frac{z}{z-1} + \frac{8}{15}\frac{z}{z+2}$$

$$y[n] = 1.8(0.5)^n u[n] - 1.\overline{33}\, u[n] + 0.5\overline{33}(-2)^n u[n]$$

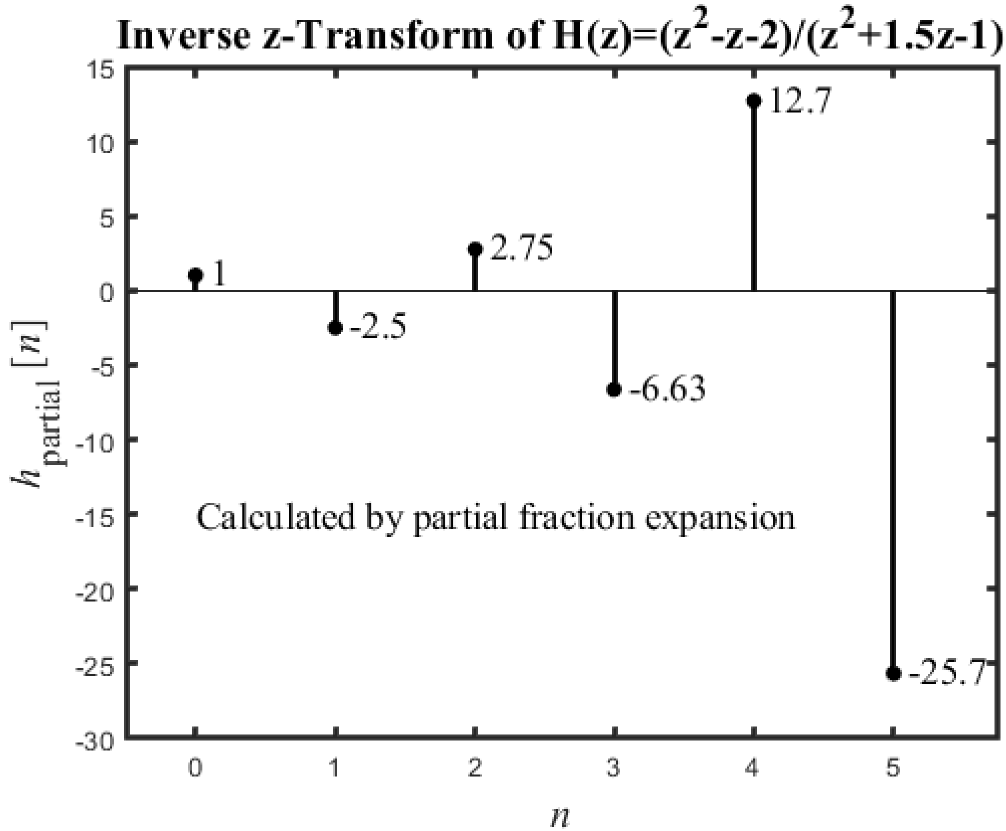
$\Rightarrow$ NOT Stable $(-2)^n$ term again

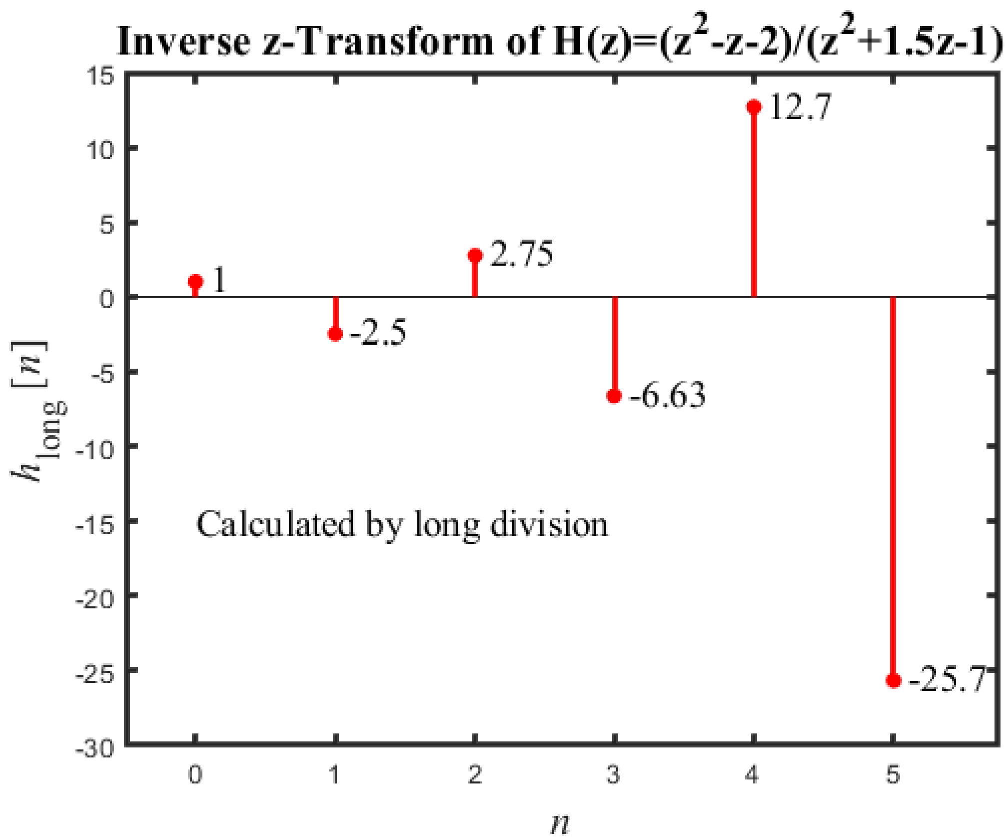
e) See attached pages

part a) plots are identical

part b) plots are identical

e) part a)





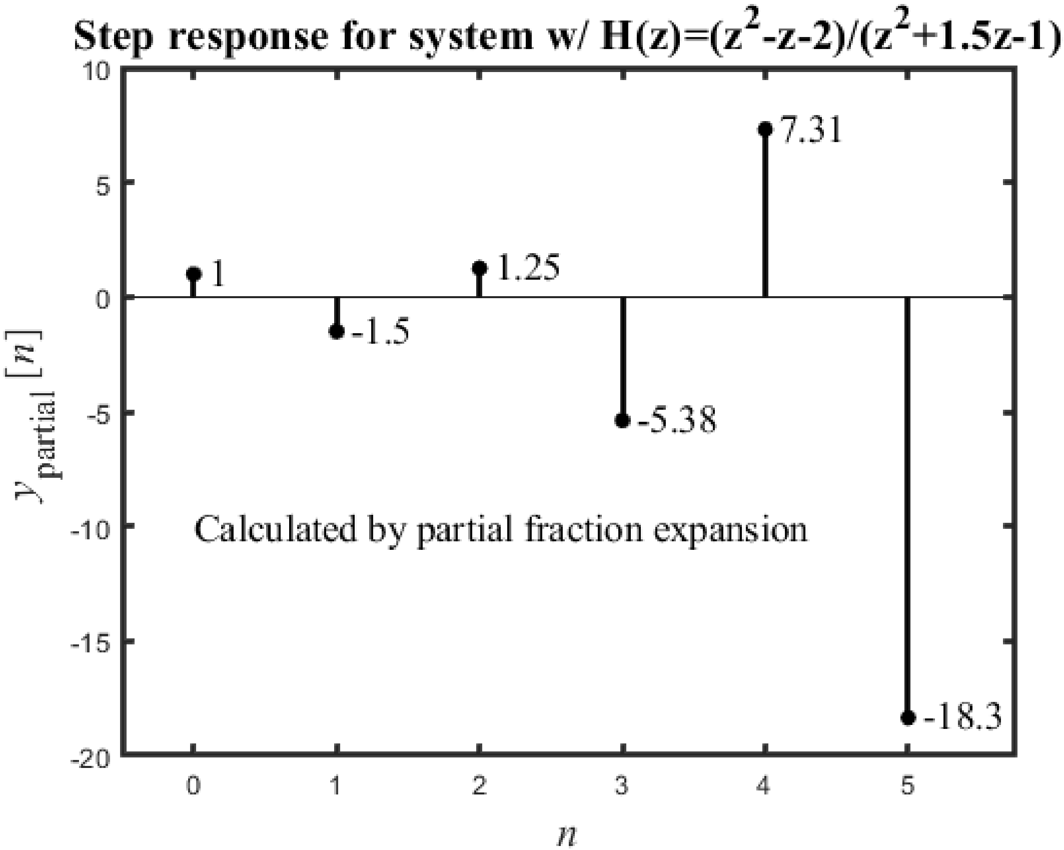
- Agree!

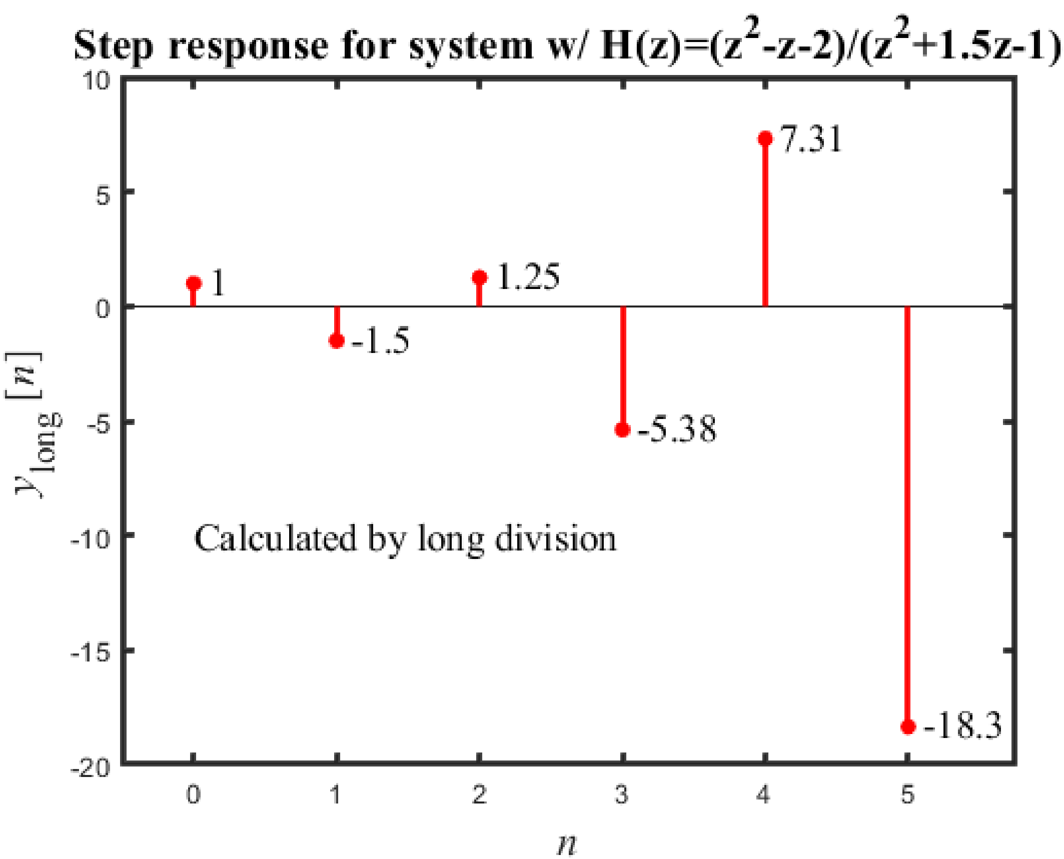
e) part a) m-file code

```
% p7_24ae.m
% Plot unit pulse response for
%   H(z)=(z^2-z-2)/(z^2+1.5z-1)
% using long division and by partial fractions solution
% h[n] = 2del[n] - 1.8(0.5)^n u[n] + 0.8(-2)^n u[n]
%
clc;clear;close all;
num = [1,-1,-2];     % Input coefficients of numerator polynomial
den = [1,1.5,-1];    % Input coefficients of denominator polynomial
% Calculate first 6 values of h[n] by long division
hlong = dimpulse(num,den,6);  n=0:5;
for l=1:length(hlong),  % partial fractions solution
    k=l-1;
    if(k==0),
        h_part(l) = 2 - 1.8*(0.5)^k + 0.8*(-2)^k;
    else
        h_part(l)= -1.8*(0.5)^k + 0.8*(-2)^k;
    end
end
stem(n,h_part,'k.','linewidth',2,'markersize',20),
axis([-0.5 5.75 -30 15]),
ylabel('{\ith}_{partial }[{\itn}]','fontsize',16,'fontname','times'),
xlabel('\itn','fontsize',16,'fontname','times'),
title('Inverse z-Transform of H(z)=(z^2-z-2)/(z^2+1.5z-1)',...
    'fontsize',16,'fontname','times'),
text(0,-15,'Calculated by partial fraction expansion',...
    'fontsize',16,'fontname','times')
for m=1:length(h_part),
    text(n(m)+0.1,h_part(m)+0.2,['',num2str(h_part(m),2)],...
        'horizontalalignment','left','verticalalignment','middle')
end
figure, stem(n,hlong,'r.','linewidth',2,'markersize',20),
axis([-0.5 5.75 -30 15]),
ylabel('{\ith}_{long }[{\itn}]','fontsize',16,'fontname','times'),
xlabel('\itn','fontsize',16,'fontname','times'),
title('Inverse z-Transform of H(z)=(z^2-z-2)/(z^2+1.5z-1)',...
    'fontsize',16,'fontname','times'),
text(0,-15,'Calculated by long division',...
    'fontsize',16,'fontname','times')
for m=1:length(hlong),

text(n(m)+0.1,hlong(m)+0.2,['',num2str(hlong(m),2)],'horizontalalignment',...
        'left','verticalalignment','middle')
end
set(findobj('type','line'),'linewidth',1.5,'markersize',18)
set(findobj('type','axes'),'linewidth',2)
set(findobj('type','text'),'fontsize',14,'fontname','times')
```

e) part b)





- Agree!

e) part b) m-file code

```
% p7_24be.m
% Plot unit step response [X(z)=z/(z-1)] for
%   H(z)=(z^2-z-2)/(z^2+1.5z-1) where
%   Y(z) =H(z)X(z) = (z^3-z^2-2z+0)/(z^3+0.5z^2-2.5z+1)
% using long division and by partial fractions solution
% y[n] = 1.8(0.5)^n u[n] + 0.8(-2)^n u[n]
%
clc;clear;close all;
num = [1,-1,-2,0];   % Input coefficients of numerator polynomial
den = [1,0.5,-2.5,1];   % Input coefficients of denominator polynomial
% Calculate first 6 values of h[n] by long division
ylong = dimpulse(num,den,6);  n=0:5;
y_part = 1.8*(0.5).^n - 4/3 + 8/15*(-2).^n;
stem(n,y_part,'k.','linewidth',2,'markersize',20),
axis([-0.5 5.75 -20 10]),
ylabel('{\ity}_{partial }[{\itn}]','fontsize',16,'fontname','times'),
xlabel('\itn','fontsize',16,'fontname','times'),
title('Step response for system w/ H(z)=(z^2-z-2)/(z^2+1.5z-1)',...
    'fontsize',16,'fontname','times'),
text(0,-10,'Calculated by partial fraction expansion',...
    'fontsize',16,'fontname','times')
for m=1:length(n),
    text(n(m)+0.1,y_part(m)+0.1,['',num2str(y_part(m),2)],...
        'horizontalalignment','left','verticalalignment','middle')
end
figure, stem(n,ylong,'r.','linewidth',2,'markersize',20),
axis([-0.5 5.75 -20 10]),
ylabel('{\ity}_{long }[{\itn}]','fontsize',16,'fontname','times'),
xlabel('\itn','fontsize',16,'fontname','times'),
title('Step response for system w/ H(z)=(z^2-z-2)/(z^2+1.5z-1)',...
    'fontsize',16,'fontname','times'),
text(0,-10,'Calculated by long division',...
    'fontsize',16,'fontname','times')
for m=1:length(n),
 text(n(m)+0.1,ylong(m)+0.1,['',num2str(ylong(m),2)],'horizontalalignment',...
        'left','verticalalignment','middle')
end
set(findobj('type','line'),'linewidth',1.5,'markersize',18)
set(findobj('type','axes'),'linewidth',2)
set(findobj('type','text'),'fontsize',14,'fontname','times')
```