7.11 Find the inverse z-transform $x[n]$ of the transforms that follow. Determine $x[n]$ for all n.

(e) $X(z) = \dfrac{2z+1}{z(10z^2 - z - 2)}$

$$\frac{X(z)}{z} = \frac{2z+1}{10z^2(z^2-0.1z-0.2)} = \frac{0.2z+0.1}{z^2(z+0.4)(z-0.5)}$$

← repeated pole

$$= \frac{C_0}{z} + \frac{C_1}{z^2} + \frac{C_2}{z+0.4} + \frac{C_3}{z-0.5}$$

Find residues

$$C_1 = \left\{ z^2 \frac{X(z)}{z} \right\}\Big|_{z=0} = \frac{0.2(0)+0.1}{(0+0.4)(0-0.5)} = -\tfrac{1}{2} = \underline{-0.5}$$

$$C_0 = \left\{ \frac{d}{dz}\left(z^2 \frac{X(z)}{z} \right) \right\}\Big|_{z=0} = \frac{d}{dz}\left(\frac{0.2z+0.1}{z^2-0.1z-0.2} \right)\Big|_{z=0}$$

$$= \frac{(z^2-0.1z-0.2)(0.2)-(0.2z+0.1)(2z-0.1)}{[z^2-0.1z-0.2]^2}\Bigg|_{z=0} = \underline{-0.75}$$

$$C_2 = \left\{ (z+0.4)\frac{X(z)}{z} \right\}\Big|_{z=-0.4} = \frac{0.2(-0.4)+0.1}{(-0.4)^2(-0.4-0.5)} = \underline{-0.13\overline{88}}$$

$$C_3 = \left\{ (z-0.5)\frac{X(z)}{z} \right\}\Big|_{z=0.5} = \frac{0.2(0.5)+0.1}{(0.5)^2(0.5+0.4)} = 8/9 = \underline{0.\overline{88}}$$

$$X(z) = -0.75 - 0.5\frac{1}{z} + \frac{-0.13\overline{88}\,z}{z+0.4} + \frac{0.\overline{88}\,z}{z-0.5}$$

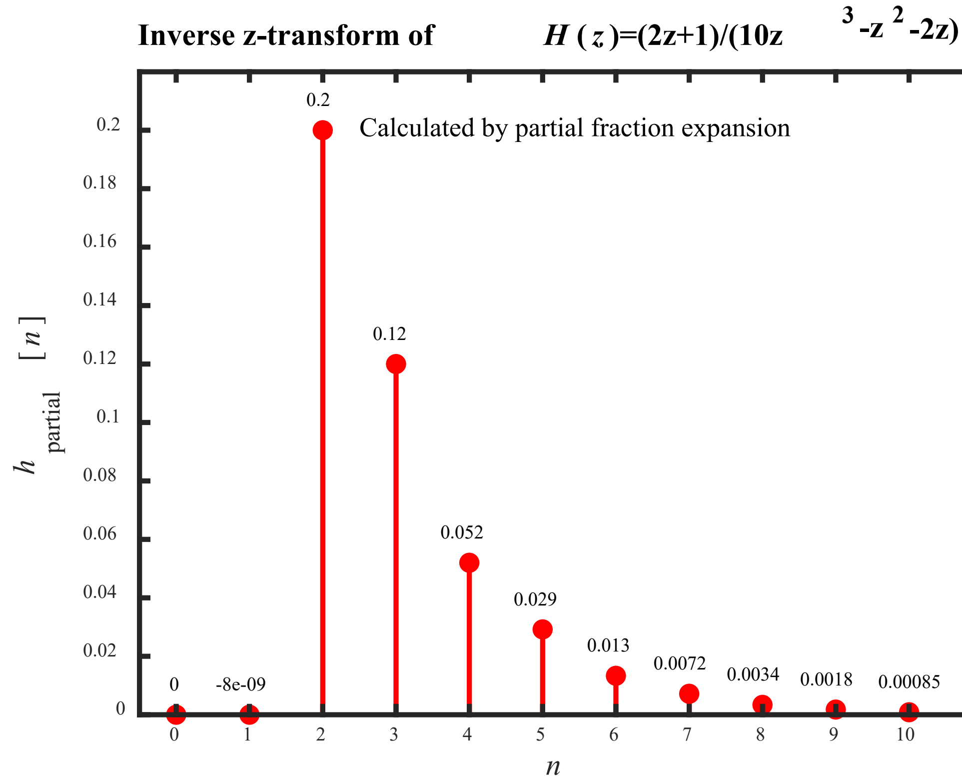
Using Linearity property, $\delta[n] \leftrightarrow 1$, $\delta[n-1] \leftrightarrow z^{-1}$,

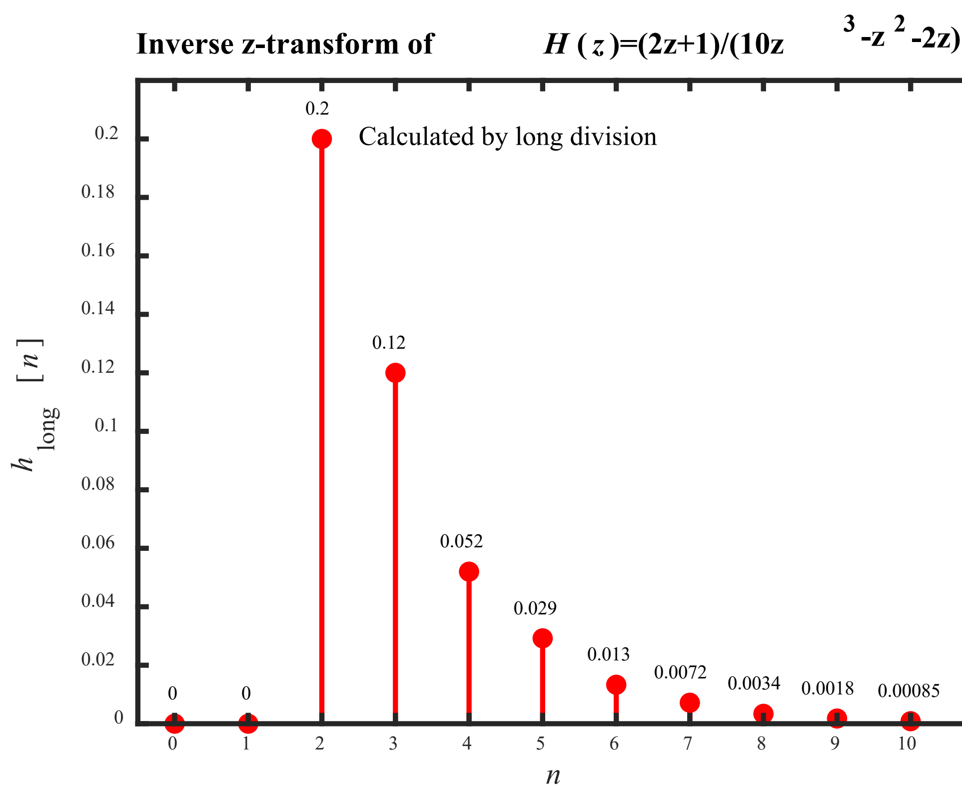
and $a^n u[n] \leftrightarrow \frac{z}{z-a}$ from Table 7.3

$$\underline{\underline{x[n] = -0.75\,\delta[n] - 0.5\,\delta[n-1] - 0.13\overline{88}(-0.4)^n u[n] + 0.\overline{88}(0.5)^n u[n]}}$$

Using MATLAB, plot the partial fractions solution for $0 \leq n \leq 10$ and on another stem plot show the answer found using long division (Hint: dimpulse.m function). Label stems. For each case, place the partial fractions solution plot on the top and the long division answer plot on the bottom of the same page. Attach m-files. Are the plots the same?

```
% Problem 7.11e (p7_11e.m)
% EE 313 Signals and Systems, your name
% Show inverse z-Transform solution to H(z)=(2z+1)/(10z^3-z^2-2z)
% using partial fractions expansion & long division for 0 <= n <=10.
clc; clear; close all;
num = [2,1]; den = [10,-1,-2,0]; %Coeff. of numerator & denominator polynomial
hlong = dimpulse(num,den,11);  % Calculate first 11 values of h[n]
n = 0:10; hpart = 0*n;
for l=1:length(n),
    k = l - 1;
    if(k==0),
        hpart(l) = -0.75-0.13888888 + 0.88888888;
    else
        if(k==1),
            hpart(l) = -0.5 - 0.13888888*(-0.4) + 0.88888888*(0.5);
        else
            hpart(l) = -0.13888888*(-0.4)^k + 0.88888888*(0.5)^k;
        end
    end
end
stem(n,hlong,'r.','linewidth',2,'markersize',20), axis([-0.5 10.9 0.0 0.22]),
ylabel('{\ith}_{long }[{\itn}]','fontsize',16,'fontname','times'),
xlabel('{\itn}','fontsize',16,'fontname','times'),
title('Inverse z-transform of {\itH}({\itz})=(2z+1)/(10z^3-z^2-2z)',...
    'fontsize',16,'fontname','times'),
text(2.5,0.2,'Calculated by long division','fontsize',16,'fontname','times')
for m=1:length(hlong),
    text(n(m),hlong(m)+0.003,['',num2str(hlong(m),2)],...
        'horizontalalignment','center','verticalalignment','bottom'),
end
figure, stem(n,hpart,'r.','linewidth',2,'markersize',20),
axis([-0.5 10.9 0.0 0.22]),
ylabel('{\ith}_{partial }[{\itn}]','fontsize',16,'fontname','times'),
xlabel('{\itn}','fontsize',16,'fontname','times'),
title('Inverse z-transform of {\itH}({\itz})=(2z+1)/(10z^3-z^2-2z)',...
    'fontsize',16,'fontname','times'),
text(2.5,0.2,'Calculated by partial fraction
expansion','fontsize',16,'fontname','times')
for m=1:length(hpart),
    text(n(m),hpart(m)+0.003,['',num2str(hpart(m),2)],...
        'horizontalalignment','center','verticalalignment','bottom'),
end
set(findobj('type','line'),'linewidth',1.5,'markersize',18)
set(findobj('type','axes'),'linewidth',2,'fontname','times')
set(findobj('type','text'),'fontsize',10,'fontname','times')
```





➢ **The plots are the same!**