7.4 Using the transform pairs in Table 7.3 and the properties of the z-transform in Table 7.2, determine the z-transform of the following discrete-time signals:

(b) $x[n] = (\sin^2 \omega n)u[n]$

Use Trigonometric identity $\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$

to get

$$x[n] = \tfrac{1}{2}\, u[n] - \tfrac{1}{2}\cos(2\omega n)\, u[n]$$

Now, use the linearity property and Table 7.3

transform pairs

$$u[n] \leftrightarrow \frac{z}{z-1}$$

$$(\cos \Omega n)u[n] \leftrightarrow \frac{z^2 - (\cos\Omega)z}{z^2 - (2\cos\Omega)z + 1}$$

to get

$$\underline{\underline{X(z) = \frac{0.5z}{z-1} - 0.5\,\frac{z^2 - (\cos 2\omega)z}{z^2 - 2\cos 2\omega\, z + 1}}}$$

or

$$\underline{\underline{X(z) = \frac{0.5(1-\cos 2\omega)z^2 + 0.5(1-\cos 2\omega)z}{z^3 - (1+2\cos 2\omega)z^2 + (1+2\cos 2\omega)z - 1}}}$$

7.4 Using the transform pairs in Table 7.3 and the properties of the z-transform in Table 7.2, determine the z-transform of the following discrete-time signals:

(f) $x[n] = ne^{-bn}(\sin \omega n)u[n]$

$$x[n] = n\,(e^{-b})^n \sin(\omega n)\, u[n]$$

Use Table 7.3 transform pair

$$a^n \sin(\Omega n)\, u[n] \longleftrightarrow \frac{a \sin(\Omega) z}{z^2 - 2a\cos(\Omega) z + a^2}$$

and the Table 7.2 property

$$n\,x[n] \longleftrightarrow -z \frac{dX(z)}{dz}$$

$$X(z) = -z \frac{d}{dz}\left[\frac{e^{-b} \sin(\omega) z}{z^2 - 2e^{-b}\cos(\omega) z + e^{-2b}} \right]$$

$$= -z \left[\frac{(z^2 - 2e^{-b}\cos(\omega) z + e^{-2b})e^{-b}\sin(\omega) - e^{-b}\sin(\omega) z (2z - 2e^{-b}\cos(\omega))}{(z^2 - 2e^{-b}\cos(\omega) z + e^{-2b})^2} \right]$$

$$= -z \left[\frac{(e^{-b}\sin\omega - 2e^{-b}\sin\omega) z^2 + (-2e^{-2b}\cos\omega\,\cancel{\sin\omega + 2e^{-2b}}\cos\omega\sin\omega) z + e^{-3b}\sin\omega}{(z^2 - 2e^{-b}\cos(\omega) z + e^{-2b})^2} \right]$$

$$X(z) = \frac{e^{-b}\sin(\omega) z^3 - e^{-3b}\sin(\omega) z}{(z^2 - 2e^{-b}\cos(\omega) z + e^{-2b})^2}$$

$$X(z) = \frac{e^{-b}\sin(\omega) z\,(z^2 - e^{-2b})}{(z^2 - 2e^{-b}\cos(\omega) z + e^{-2b})^2}$$