7.2 A discrete-time signal $x[n]$ has z-transform $X(z) = \dfrac{z}{8z^2 - 2z - 1}$

Determine the z-transform $V(z)$ of the following signals:

(a) $v[n] = x[n-4]u[n-4]$ **(c)** $v[n] = \cos(2n)x[n]$

(e) $v[n] = n^2 x[n]$

a) From Table 7.2, use $x[n-q]u[n-q] \leftrightarrow z^{-q}X(z)$

Here $q=4$, therefore

$$V(z) = z^{-4}\frac{z}{8z^2-2z-1} = \frac{1}{z^3(8z^2-2z-1)}$$

$$\underline{\underline{V(z) = \frac{1}{8z^5-2z^4-z^3}}}$$

c) From Table 7.2, use

$$\cos(rn)x[n] \leftrightarrow \tfrac{1}{2}\left[X(e^{jr}z) + X(e^{-jr}z)\right]$$

$$V(z) = \tfrac{1}{2}\left[\frac{e^{jr}z}{8(e^{jr}z)^2 - 2e^{jr}z - 1} + \frac{e^{-jr}z}{8(e^{-jr}z)^2 - 2e^{-jr}z - 1}\right]\Bigg|_{r=2}$$

$$\underline{\underline{V(z) = \tfrac{1}{2}\left[\frac{e^{j2}z}{8e^{j4}z^2 - 2e^{j2}z - 1} + \frac{e^{-j2}z}{8e^{-j4}z^2 - 2e^{-j2}z - 1}\right]}}$$

e) use Table 7.2, $n^2x[n] \leftrightarrow z\frac{dX(z)}{dz} + z^2\frac{d^2X(z)}{dz^2}$

$$\frac{d}{dz}\left[\frac{z}{8z^2-2z-1}\right] = \frac{(8z^2-2z-1)(1) - z(16z-2)}{(8z^2-2z-1)^2} = \frac{-8z^2-1}{(8z^2-2z-1)^2}$$

$$\frac{d^2}{dz^2}\left[\frac{z}{8z^2-2z-1}\right] = \frac{(8z^2-2z-1)^2(-16z) - (-8z^2-1)\,2(8z^2-2z-1)(16z-2)}{(8z^2-2z-1)^4}$$

$$= \frac{(8z^2-2z-1)(-16z) + (8z^2+1)(32z-4)}{(8z^2-2z-1)^3}$$

$$V(z) = \frac{-8z^3-z}{(8z^2-2z-1)^2} + \frac{z^2\left[-128z^3+32z^2+16z+256z^3-32z^2+32z-4\right]}{(8z^2-2z-1)^3}$$

$$\underline{\underline{V(z) = \frac{64z^5+16z^4+48z^3-2z^2+z}{(8z^2-2z-1)^3}}}$$