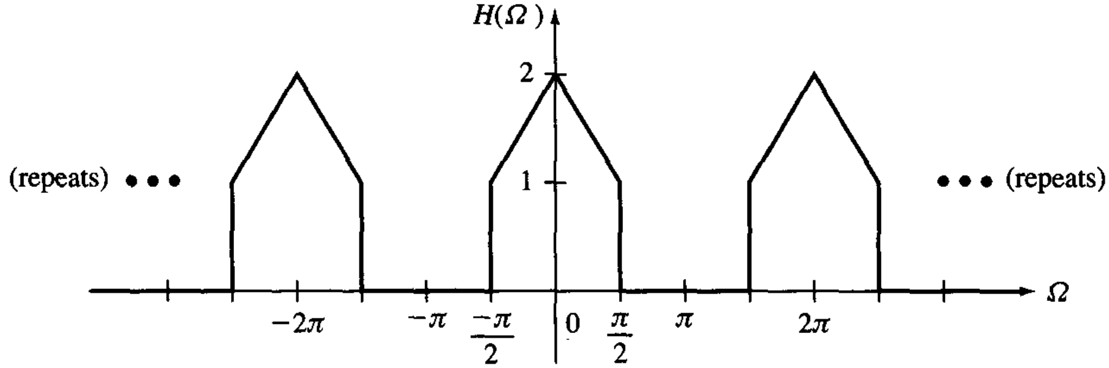
5.40 A linear time-invariant discrete-time system has the frequency response function $H(\Omega)$ shown in Figure P5.40.



(a) Determine the unit-pulse response $h[n]$ of the system.
(b) Compute the output response $y[n]$ when the input $x[n]$ is equal to $\delta[n] - \delta[n-1]$.
(c) Compute the output response $y[n]$ when the input is $x[n] = 2 + \sin(\pi n/4) + 2\sin(\pi n/2)$.
(d) Compute the output response $y[n]$ when $x[n] = \text{sinc}(n/4)$, $n = 0, \pm 1, \pm 2$.

a) w/in the $-\pi < \Omega < \pi$ range

$$H(\Omega) = p_{\pi}(\Omega) + \wedge_{\pi}(\Omega)$$

From Table 4.1, $\frac{B}{\pi}\text{sinc}\left(\frac{B}{\pi}n\right) \Leftrightarrow \sum_{k=-\infty}^{\infty} p_{2B}(\Omega + 2\pi k)$

From Table 4.2, If $x[n] \Leftrightarrow X(\Omega)$ and $\gamma(t) \Leftrightarrow X(\omega)p_{2\pi}(\omega)$,
then $x[n] = \gamma(t)\big|_{t=n}$

and use $\frac{\tau}{2}\text{sinc}^2\left(\frac{\tau t}{4\pi}\right) \Leftrightarrow 2\pi \wedge_{\tau}(\omega)$ from Table 3.2

to get

$$h[n] = \frac{\pi/2}{\pi}\text{sinc}\left(\frac{\pi/2}{\pi}n\right) + \frac{\pi}{4\pi}\text{sinc}^2\left(\frac{\pi t}{4\pi}\right)\bigg|_{t=n}$$

$$\underline{\underline{h[n] = \tfrac{1}{2}\text{sinc}(n/2) + \tfrac{1}{4}\text{sinc}^2(n/4) \quad -\infty < n < \infty}}$$

b) Since $y[n] = h[n]$ when $x[n] = \delta[n]$

and using linearity, when $x[n] = \delta[n] - \delta[n-1]$

$$y[n] = h[n] - h[n-1]$$

$$\underline{\underline{y[n] = \tfrac{1}{2}\,\text{sinc}(n/2) + \tfrac{1}{4}\,\text{sinc}^2(n/4) - \tfrac{1}{2}\,\text{sinc}\left(\tfrac{n-1}{2}\right) - \tfrac{1}{4}\,\text{sinc}^2\left(\tfrac{n-1}{4}\right)}}$$

$-\infty < n < \infty$

c) Find $y[n]$ for $x[n] = 2 + \sin(\frac{\pi}{4}n) + 2\sin(\frac{\pi}{2}n)$.

Use (5.65), $y[n] = A|H(\Omega_0)|\cos(\Omega_0 n + \theta + \measuredangle H(\Omega_0))$

for $x[n] = A\cos(\Omega_0 n + \theta)$, and linearity to get

$$y[n] = 2|H(0)| + (1)|H(\pi/4)|\sin(\tfrac{\pi}{4}n + \measuredangle H(\pi/4)) + (2)|H(\pi/2)|\sin(\tfrac{\pi}{2}n + \measuredangle H(\pi/2))$$

$$= 2(2) + (1)(1.5)\sin(\pi/4\, n + 0) + (2)(1)\sin(\pi/2\, n + 0)$$

$$\underline{\underline{y[n] = 4 + 1.5\sin(\pi/4\, n) + 2\sin(\pi/2\, n)}} \quad -\infty < n < \infty$$

Note: (5.65) works the same for the sine function as the cosine function as they are the same thing w/ a $\pi/2$ phase shift.

d) Find $y[n]$ when $x[n] = \text{sinc}(n/4)$ $n = 0, \pm 1, \ldots$

From Table 4.1, $\frac{B}{\pi}\text{sinc}\left(\frac{B}{\pi}n\right) \longleftrightarrow \sum_{k=-\infty}^{\infty} p_{2B}(\Omega + 2\pi k)$

and use linearity

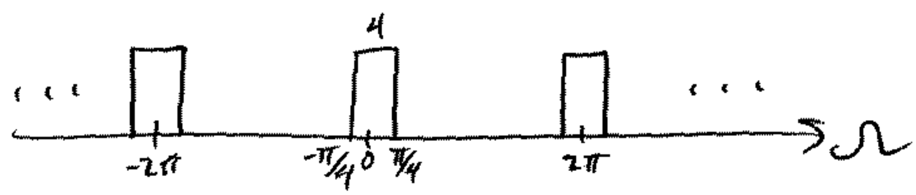
$$\frac{B}{\pi}n = \frac{n}{4} \Rightarrow B = \pi/4$$

So,

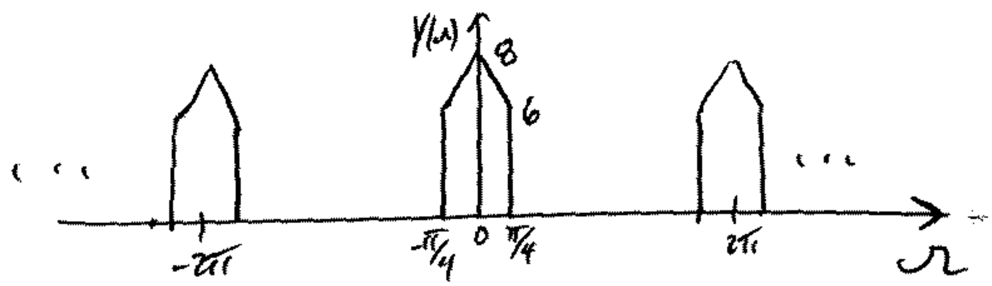
$$X(\Omega) = \frac{\pi}{(\pi/4)} \sum_{k=-\infty}^{\infty} p_{2(\pi/4)}(\Omega + 2\pi k)$$

$$= 4 \sum_{k=-\infty}^{\infty} p_{\pi/2}(\Omega + 2\pi k)$$

plot



Since $Y(\Omega) = X(\Omega)H(\Omega)$, we get



or $Y(\Omega) = 6\, p_{\pi/2}(\Omega) + 2\, \Lambda_{\pi/2}(\Omega)$ for $-\pi < \Omega < \pi$

and

$$y[n] = 6\,\frac{\pi/4}{\pi}\,\text{sinc}\left(\frac{\pi/4}{\pi}n\right) + 2\,\frac{\pi/2}{4\pi}\,\text{sinc}^2\left(\frac{\pi/2\; t}{4\pi}\right)\Big|_{t=n}$$

$$\underline{\underline{y[n] = 1.5\,\text{sinc}(n/4) + \frac{1}{4}\,\text{sinc}^2\left(\frac{n}{8}\right) \quad -\infty < n < \infty}}$$