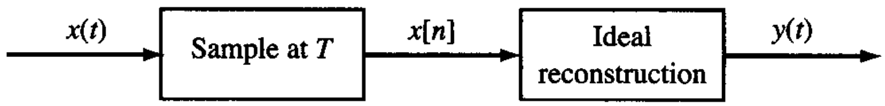
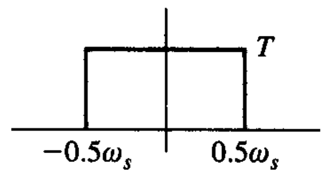
5.35 Repeat Problem 5.34 for $x(t) = 2 + \cos(50\pi t)$ and $T = 0.025$ sec.

5.34 Consider the following sampling and reconstruction configuration:



You can find the output $y(t)$ of the ideal reconstruction by sending the sampled signal $x_s(t) = x(t)p(t)$ through an ideal lowpass filter with the frequency response function



Let $x(t) = 2 + \cos(50\pi t)$ and $T = 0.01$ sec.
(a) Draw $|X_s(\omega)|$, where $x_s(t) = x(t)p(t)$. Determine if aliasing occurs.
(b) Determine the expression for $y(t)$.
(c) Determine an expression for $x[n]$.

a) Using Table 3.2, $1 \leftrightarrow 2\pi\delta(\omega)$

$$\cos\omega_0 t \leftrightarrow \pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$$

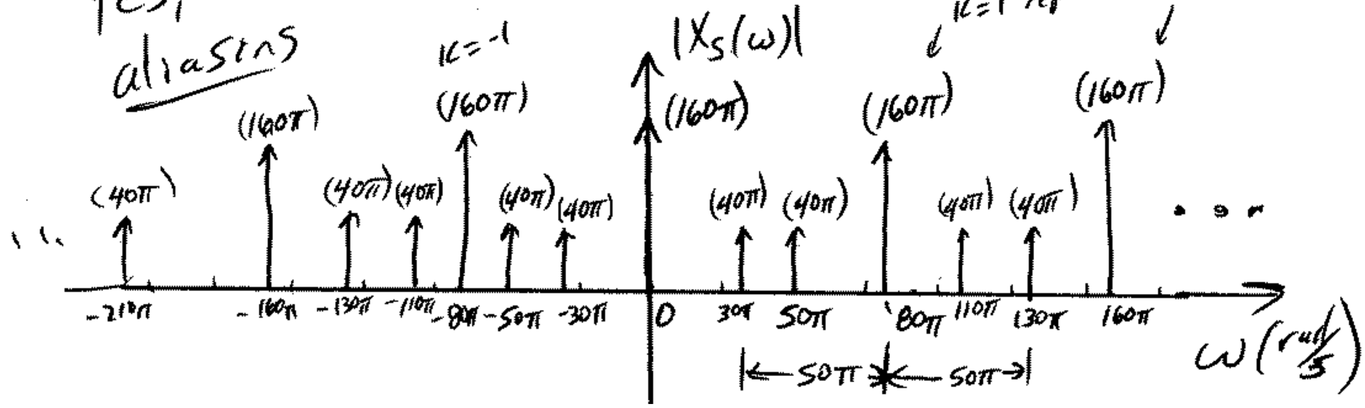
and Linearity property

$$X(\omega) = 4\pi\delta(\omega) + \pi\delta(\omega+50\pi) + \pi\delta(\omega-50\pi)$$

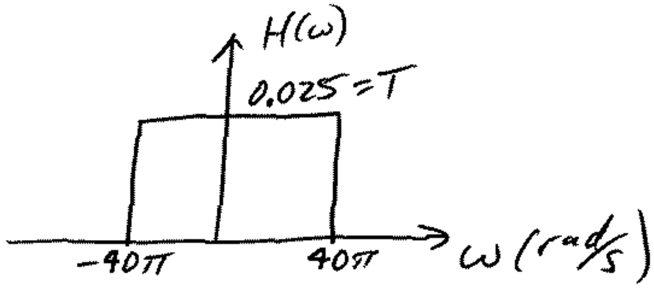
Next, use (5.51) $X_s(\omega) = \sum_{k=-\infty}^{\infty} \frac{1}{T} X(\omega - k\omega_s)$

where $\omega_s = 2\pi/T = \frac{2\pi}{0.025} = 80\pi$ rad/s and $1/T = \frac{1}{0.025} = 40s^{-1}$

Yes, there is aliasing



b) In this case, the ideal LP filter has response



Therefore, $Y(\omega) = H(\omega)\,X(\omega)$

$$= 0.025\left[40\pi\,\delta(\omega+30\pi) + 160\pi\,\delta(\omega) + 40\pi\,\delta(\omega-30\pi)\right]$$

$$= \pi\,\delta(\omega+30\pi) + 4\pi\,\delta(\omega) + \pi\,\delta(\omega-30\pi)$$

and $\underline{y(t) = 2 + \cos(30\pi t) \quad -\infty < t < \infty}$

Note: $y(t) \neq x(t)$ due to aliasing!

c) Find $x[n] = x(t = nT)$

$$= 2 + \cos(50\pi n \overset{0.025}{T})$$

$\underline{x[n] = 2 + \cos(1.25\pi n) \quad -\infty < n < \infty}$

or using Trig IDs

$$= 2 + \cos(-0.75\pi n)$$

$\underline{x[n] = 2 + \cos(0.75\pi n) \quad -\infty < n < \infty}$