5.29 Consider the system in Figure P5.29, where $p(t)$ is an impulse train with period T and $H(\omega) = T p_2(\omega)$. Compute $y(t)$ when

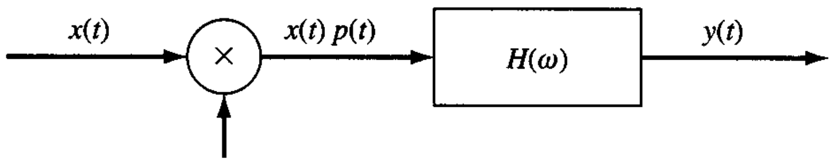


FIGURE P5.29

(a) $x(t) = \text{sinc}^2(t/2\pi)$ for $-\infty < t < \infty, T = \pi$
(b) $x(t) = \text{sinc}^2(t/2\pi)$ for $-\infty < t < \infty, T = 2\pi$
(c) For (a) and (b), compare the plots of $x(t)$ and the corresponding $y(t)$.
(d) Repeat part (a), using the interpolation formula to solve for $y(t)$, and plot your results with n ranging from $n = -5$ to $n = 5$.

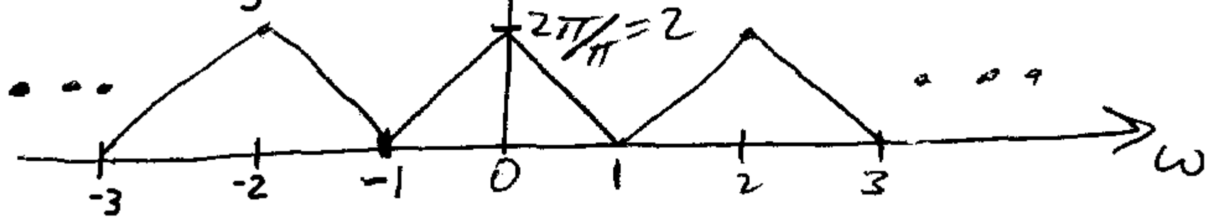
a) $x(t) = \text{sinc}^2(t/2\pi) = \frac{2}{2}\,\text{sinc}^2\left(\frac{2t}{4\pi}\right)$ Table 4.2

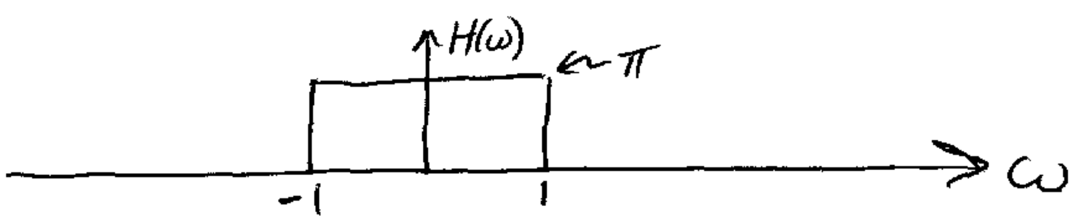
$$X(\omega) = 2\pi\left(1 - \frac{2|\omega|}{2}\right) p_2(\omega) = 2\pi\Lambda_2(\omega)$$

If $T = \pi$, $\omega_s = \frac{2\pi}{T} = 2$

$\Rightarrow$ No aliasing

$$X_s(\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(\omega - k\omega_s)$$





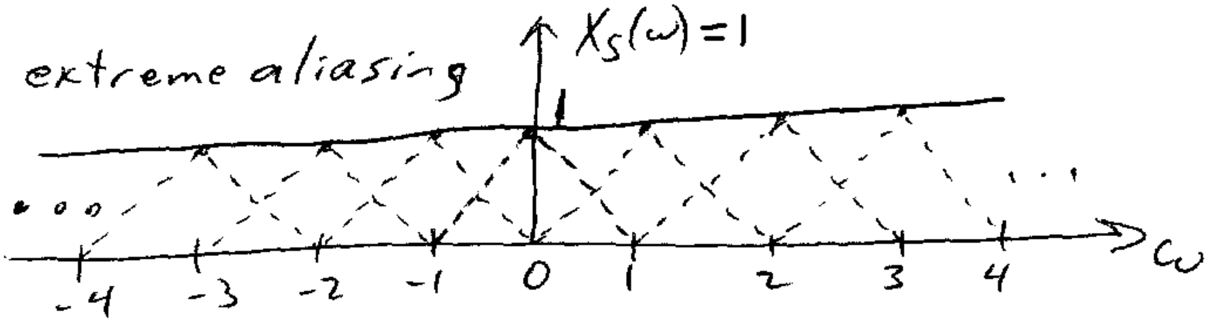
$$Y(\omega) = X_s(\omega) H(\omega) = 2\pi\left(1 - \frac{2|\omega|}{2}\right) p_2(\omega) = X(\omega)$$

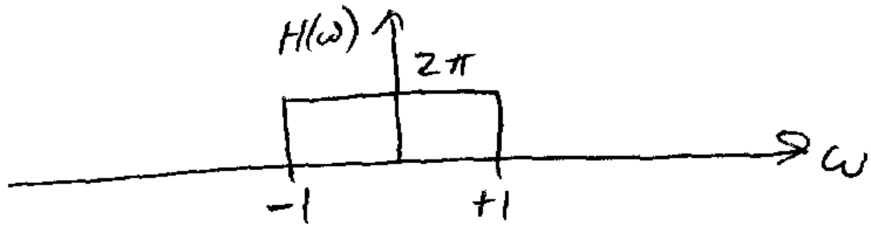
$$y(t) = x(t) = \text{sinc}^2\left(\frac{t}{2\pi}\right) \quad -\infty < t < \infty$$

b) again $X(\omega) = 2\pi\left(1 - \frac{2|\omega|}{2}\right) p_2(\omega)$

If $T = 2\pi$, $\omega_s = \frac{2\pi}{T} = 1$ and $X_s(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} X(\omega - k(1))$

→ extreme aliasing





$Y(\omega) = (1)\, 2\pi\, p_2(\omega) = 2\pi\, p_2(\omega)$

$\underline{\underline{y(t) = 2\,\text{sinc}\left(\frac{2t}{2\pi}\right) = 2\,\text{sinc}(t/\pi) \quad -\infty < t < \infty}}$

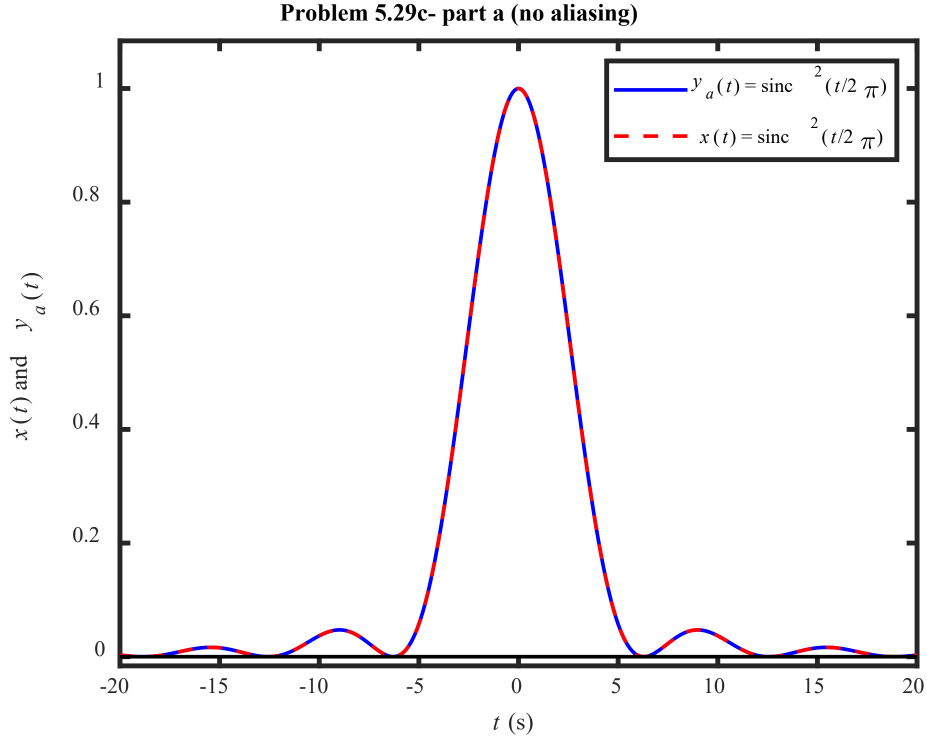
c) compare plots of $x(t)$ + $y(t)$

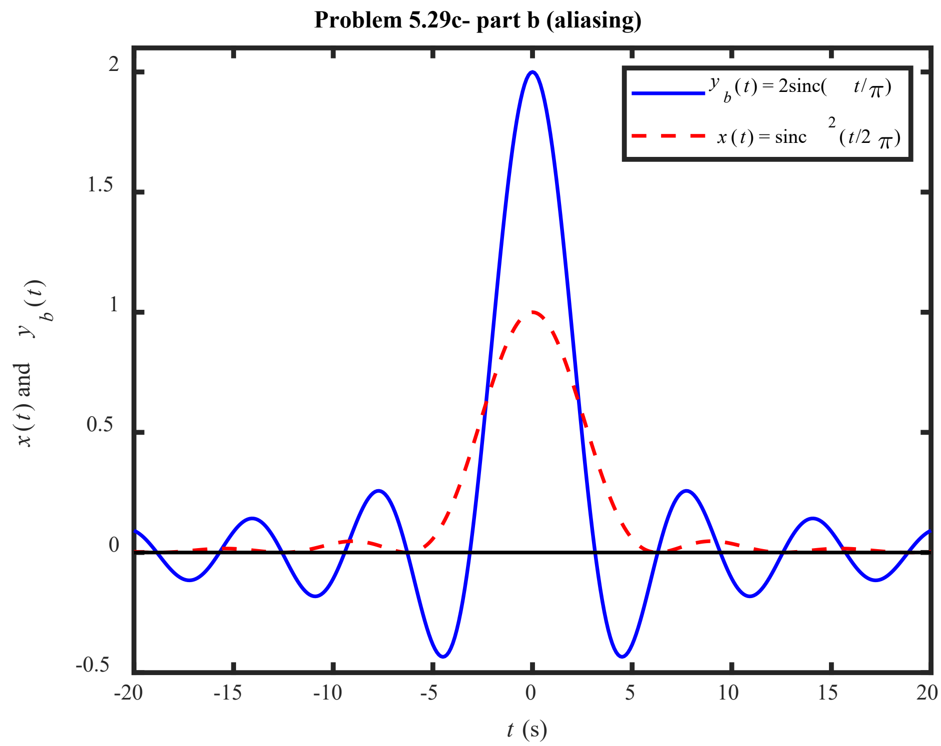
(see attached)

⇒ part a) $x(t)$ + $y(t)$ identical

⇒ part b) $x(t)$ + $y(t)$ significantly different

c)






```
% Problem 5.29c (p5_29c.m)
% Plot x(t) & y(t) for parts a) & b)
close all; clear; clc;
t = -20 : 0.1 : 20; z = 0*t;
x = sinc(t/2/pi).^2; ya = sinc(t/2/pi).^2; % input & output part a)
yb = 2*sinc(t/pi);    % output part b)
plot(t,ya,'b-',t,x,'r--',t,z,'k'),axis([-20 20 0 1.1]),
xlabel('{\itt} (s)','fontsize',16,'fontname','times'),
```

```
ylabel(['{\itx}({\itt}) and {\ity_a}({\itt})'],'fontsize',16,...
    'fontname','times'),
title('Problem 5.29c- part a (no aliasing)',...
    'fontsize',18,'fontname','times');
legend('{\ity_a}({\itt}) = sinc^2({\itt}/2\pi)',...
    ' {\itx}({\itt}) = sinc^2({\itt}/2\pi)'),
figure,plot(t,yb,'b-',t,x,'r--',t,z,'k'),axis([-20 20 -0.5 2.1]),
title('Problem 5.29c- part b (aliasing)',...
    'fontsize',18,'fontname','times');
ylabel(['{\itx}({\itt}) and {\ity_b}({\itt})'],'fontsize',16,...
    'fontname','times'),
xlabel('{\itt} (s)','fontsize',16,'fontname','times'),
legend('{\ity_b}({\itt}) = 2sinc({\itt}/\pi)',...
    ' {\itx}({\itt}) = sinc^2({\itt}/2\pi)'),
set(findobj('type','line'),'linewidth',1.5)
set(findobj('type','axes'),'linewidth',2,'fontname','times','fontsize',12)
```

d) Interpolation Formula

$$y(t) = \frac{BT}{\pi} \sum_{n=-\infty}^{\infty} x(nT)\,\mathrm{sinc}\left[\frac{B}{\pi}(t-nT)\right] \quad (5.62)$$

$$B = 1 \text{ rad/s}, \quad T = \pi \text{ (sec)}$$

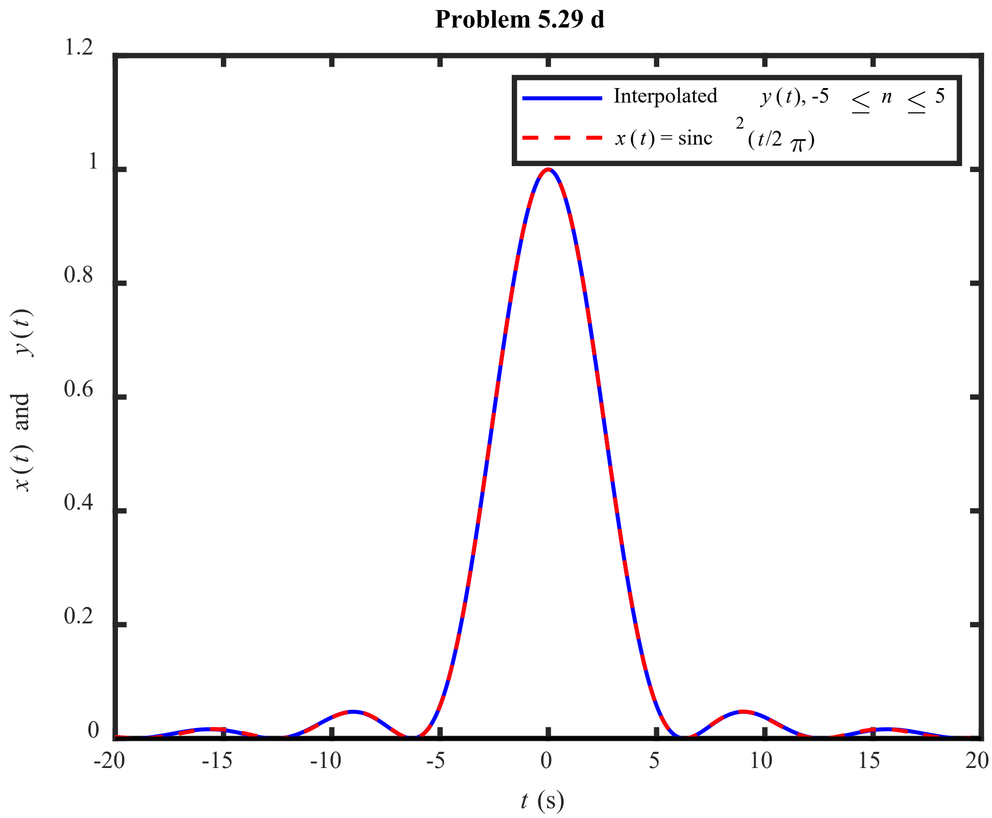
$$y(t) = \frac{1(\pi)}{\pi} \sum_{n=-\infty}^{\infty} \mathrm{sinc}^2\left(\frac{n\pi}{2\pi}\right) \mathrm{sinc}\left[\frac{1}{\pi}(t-n\pi)\right]$$

$$\underline{\underline{y(t) = \sum_{n=-\infty}^{\infty} \mathrm{sinc}^2(n/2)\,\mathrm{sinc}\left[\frac{1}{\pi}(t-n\pi)\right]}}$$

plot + m-file for $y(t)$ for $-5 \le n \le 5$ are attached, $x(t)$ is also plotted on this figure for comparison.

$\Rightarrow$ Note that $y(t)$ + $x(t)$ are nearly identical.

d) cont.




```
% Problem 5.29d (p5_29d.m)
% Plot x(t) & y(t), using interpolation formula for part a)
close all; clear; clc;
t = -20:0.1:20; z=0*t;
x = sinc(t/2/pi).^2;     % input
y = zeros(1,length(t)); % output
for n=-5:5,
    y = y + sinc(n/2)^2 * sinc((t-n*pi)/pi);
end
plot(t,y,'b-',t,x,'r--',t,z,'k'), axis([-20 20 0 1.2]),
ylabel(['{\itx}({\itt})  and  {\ity}({\itt})'],...
    'fontsize',16,'fontname','times'),
title('Problem 5.29 d','fontsize',18,'fontname','times');
xlabel('{\itt} (s)','fontsize',16,'fontname','times'),
legend(' Interpolated {\ity}({\itt}), -5 \leq {\itn} \leq 5',...
    ' {\itx}({\itt}) = sinc^2({\itt}/2\pi)'),
set(findobj('type','line'),'linewidth',1.5)
set(findobj('type','axes'),'linewidth',2,...
    'fontname','times','fontsize',12)
```