5.14 An ideal linear-phase lowpass filter has the frequency response function

$$H(\omega) = \begin{cases} e^{-j\omega}, & -2 < \omega < 2 \\ 0, & \text{all other } \omega \end{cases}$$

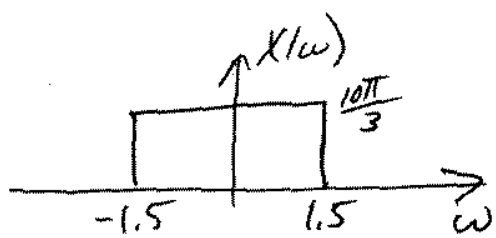
Compute the filter's output response $y(t)$ for the different inputs $x(t)$ as given next. Plot each input $x(t)$ and the corresponding output $y(t)$. Also plot the magnitude and phase functions for $X(\omega)$, $H(\omega)$, and $Y(\omega)$.

(a) $x(t) = 5\,\mathrm{sinc}(3t/2\pi), -\infty < t < \infty$

a) Use Fourier transform pair $\tau\,\mathrm{sinc}\left(\frac{\tau t}{2\pi}\right) \leftrightarrow 2\pi p_\tau(\omega)$ and linearity

↳ $\mathrm{sinc}\left(\frac{\tau t}{2\pi}\right) \leftrightarrow \frac{2\pi}{\tau} p_\tau(\omega)$

$$5\,\mathrm{sinc}\left(\frac{3t}{2\pi}\right) \leftrightarrow 5\left(\frac{2\pi}{3}\right) p_3(\omega) = \frac{10\pi}{3} p_3(\omega) = X(\omega)$$



↳ all of $X(\omega)$ gets through (with a linear phase shift).

$$Y(\omega) = X(\omega)H(\omega) = \left(\frac{10\pi}{3} p_3(\omega)\right)\left(e^{-j\omega} p_4(\omega)\right)$$

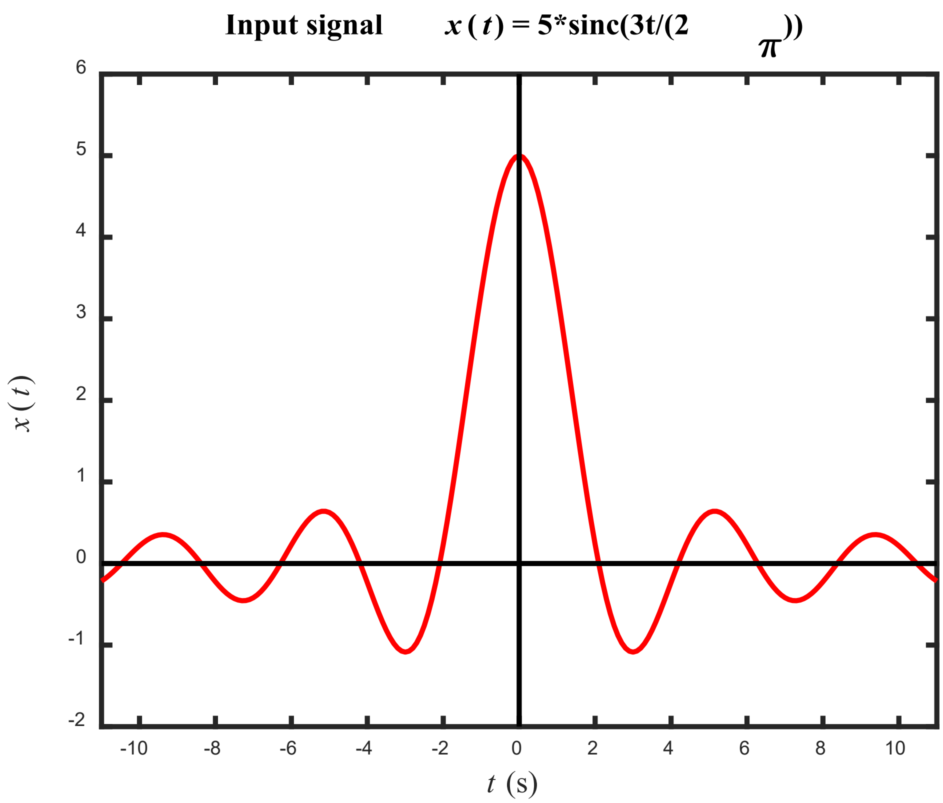
$$= \frac{10\pi}{3} e^{-j\omega} p_3(\omega)$$

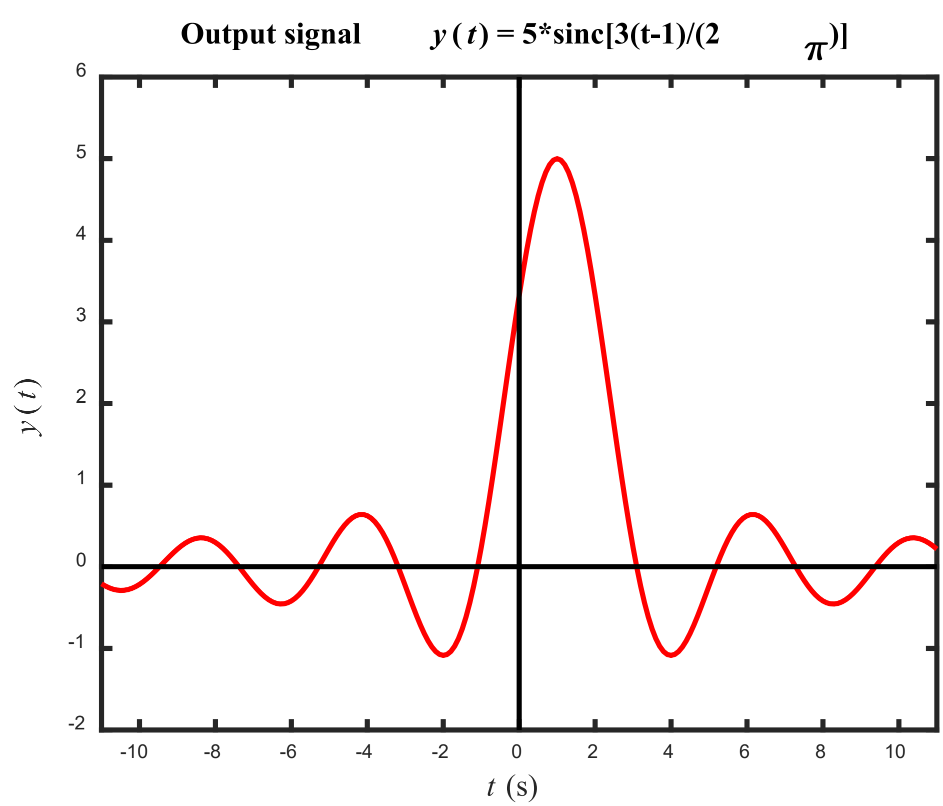
Use Time-shift property $x(t-c) \leftrightarrow X(\omega)e^{-j\omega c}$ (w/ $c=1$) and above to get

$$y(t) = 5\,\mathrm{sinc}\left(\frac{3(t-1)}{2\pi}\right) \quad -\infty < t < \infty$$

a) cont.

- Use Matlab to plot $x(t)$ and $y(t)$ for $-11 < t < 11$ s.

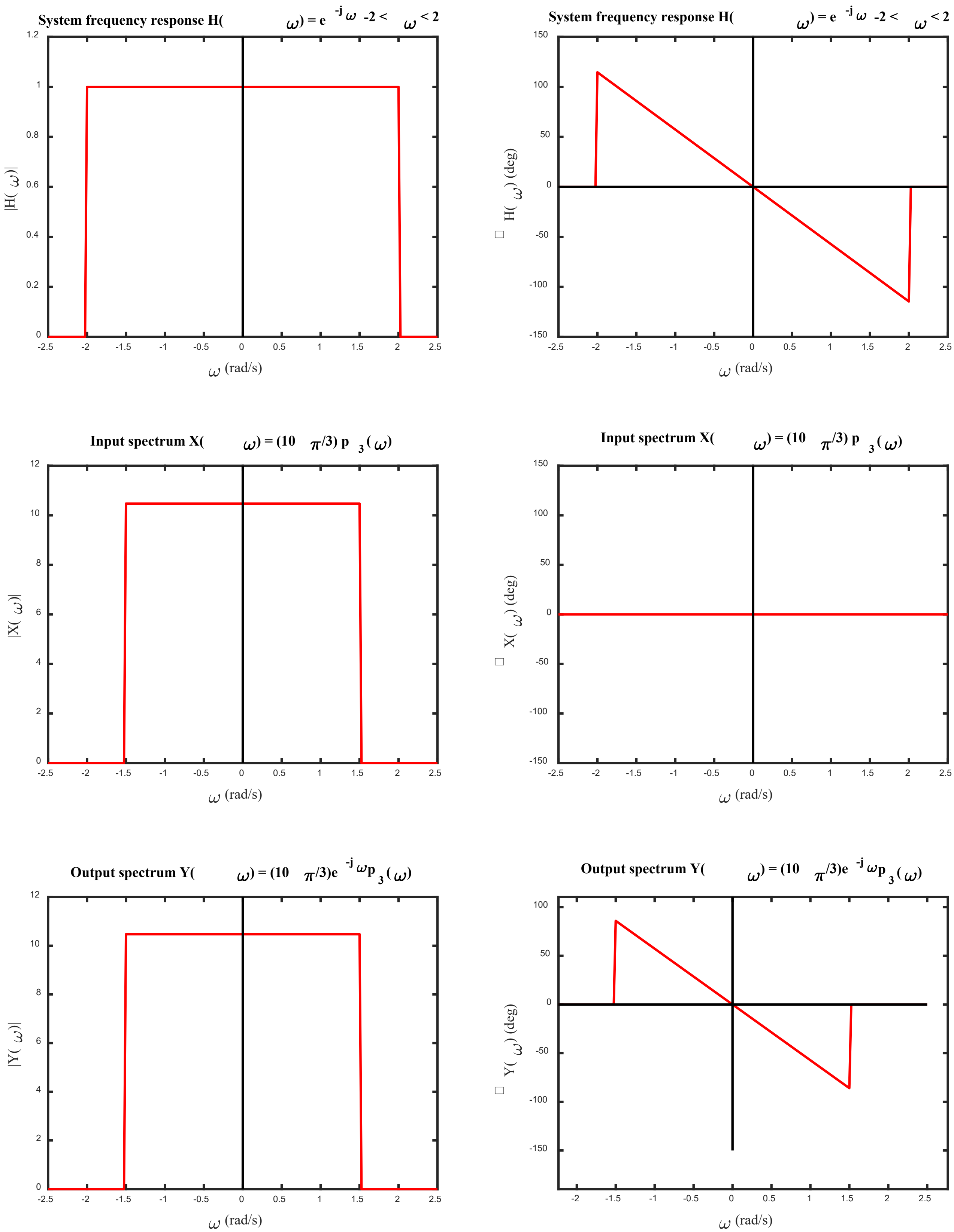




Note the 1 second time-delay from the lowpass filter.

a) cont.

- Have frequency plots range be -2.5 < ω < 2.5 rad/s.



a) cont.

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% Chapter 5 problem 5.14a (chap5_5_14a.m)
% Plot magnitude and phase spectrum of
%   H(w) = exp(-jw)*p4(w), X(w) = (10pi/3)*p3(w), and
%   Y(w) = (10pi/3)*exp(-jw)*p3(w).
% Also, plot the time-domain signals
%  x(t) = 5sinc(3t/(2pi)) and y(t) = 5sinc(3(t-1)/(2pi)).
clear; clc; close all;
w = -2.5:0.025:2.5;  % Define frequency vector
% calculate system frequency response H(w)
H = exp(-j*w);
for k = 1:length(w),
    if(abs(w(k))<-2|abs(w(k))>2),
        H(k) = 0; % limit bandwidth
    end
end
Hmag=abs(H); Hang=angle(H)*180/pi; % spectrum of H(w)
% calculate input spectrum X(w)
X = 10*pi/3*ones(1,length(w));
for k = 1:length(w),
    if(abs(w(k))<-1.5|abs(w(k))>1.5),
        X(k) = 0; % limit bandwidth
    end
end
Xmag = abs(X); Xang = angle(X)*180/pi; % spectrum of X(w)
% calculate output spectrum Y(w)
Y = X.*H;
Ymag = abs(Y); Yang = angle(Y)*180/pi; % spectrum of Y(w)
% Calculate time-domain input x(t) & output y(t)
t = -11:0.1:11;
x = 5*sinc(3*t/2/pi); y = 5*sinc(3*(t-1)/2/pi);
% Plot amplitude and phase spectrum of H(w)
plot(w,Hmag,'r-',[0 0],[0 2.5],'k-'), axis([-2.5 2.5 0 1.2]),
xlabel('\omega (rad/s)','fontsize',16,'fontname','times'),
ylabel('|H(\omega)|','fontsize',16,'fontname','times'),
title(['System frequency response ',...
    'H(\omega) = e^{-j\omega}  -2 < \omega < 2'],...
    'fontsize',16,'fontname','times'),
figure, plot(w,Hang,'r-',[-2.5 2.5],[0 0],'k-',[0 0],[-150 150],'k-'),
axis([-2.5 2.5 -150 150]),
xlabel('\omega (rad/s)','fontsize',16,'fontname','times'),
ylabel('\angle H(\omega) (deg)','fontsize',16,'fontname','times'),
title(['System frequency response ',...
    'H(\omega) = e^{-j\omega}  -2 < \omega < 2'],...
    'fontsize',16,'fontname','times'),
% Plot amplitude and phase spectrum of X(w)
figure, plot(w,Xmag,'r-',[0 0],[0 12],'k-'),
axis([-2.5 2.5 0 12]),
xlabel('\omega (rad/s)','fontsize',16,'fontname','times'),
ylabel('|X(\omega)|','fontsize',16,'fontname','times'),
title(['Input spectrum X(\omega) = (10\pi/3) p_3(\omega)'],...
    'fontsize',16,'fontname','times'),
figure, plot(w,Xang,'r-',[0 0],[-150 150],'k-'),axis([-2.5 2.5 -150 150]),
xlabel('\omega (rad/s)','fontsize',16,'fontname','times'),
ylabel('\angle X(\omega) (deg)','fontsize',16,'fontname','times'),
```

a) cont.

```
title(['Input spectrum X(\omega) = (10\pi/3) p_3(\omega)'],...
    'fontsize',16,'fontname','times'),
% Plot amplitude and phase spectrum of Y(w)
figure, plot(w,Ymag,'r-',[0 0],[0 12],'k-'),
axis([-2.5 2.5 0 12]),
xlabel('\omega (rad/s)','fontsize',16,'fontname','times'),
ylabel('|Y(\omega)|','fontsize',16,'fontname','times'),
title(['Output spectrum Y(\omega) = (10\pi/3)e^{-j\omega}p_3(\omega)'],...
    'fontsize',16,'fontname','times'),
figure,plot(w,Yang,'r-',[-2.5 2.5],[0 0],'k-',[0 0],[-150 150],'k-'),
axis([-2.5 2.5 -150 150]),
xlabel('\omega (rad/s)','fontsize',16,'fontname','times'),
ylabel('\angle Y(\omega) (deg)','fontsize',16,'fontname','times'),
title(['Output spectrum Y(\omega) = (10\pi/3)e^{-j\omega}p_3(\omega)'],...
    'fontsize',16,'fontname','times'),
% Plot time-domain input x(t)
figure, plot(t,x,'r-',[-11 11],[0 0],'k-',[0 0],[-2 6],'k-'),
axis([-11 11 -2 6]),
xlabel('{\itt} (s)','fontsize',16,'fontname','times'),
ylabel('{\itx}({\itt})','fontsize',16,'fontname','times'),
title(['Input signal {\itx}({\itt}) = 5*sinc(3t/(2\pi))'],...
    'fontsize',16,'fontname','times'),
% Plot time-domain output y(t)
figure, plot(t,y,'r-',[-11 11],[0 0],'k-',[0 0],[-2 6],'k-'),
axis([-11 11 -2 6]),
xlabel('{\itt} (s)','fontsize',16,'fontname','times'),
ylabel('{\ity}({\itt})','fontsize',16,'fontname','times'),
title(['Output signal {\ity}({\itt}) = 5*sinc[3(t-1)/(2\pi)]'],...
    'fontsize',16,'fontname','times'),
set(findobj('type','line'),'linewidth',2)
set(findobj('type','axes'),'linewidth',2)
set(findobj('text','line'),'fontname','times')
```