5.13 A linear time-invariant continuous-time system has the frequency function $H(\omega)$. It is known that the input

$$x(t) = 1 + 4\cos 2\pi t + 8\sin(3\pi t - 90°)$$

produces the response

$$y(t) = 2 - 2\sin 2\pi t$$

(a) For what values of ω is it possible to determine $H(\omega)$?

(b) Compute $H(\omega)$ for each of the values of ω determined in part (a).

a) $x(t) = 1 + 4\cos 2\pi t + 8\cos(3\pi t - 180°)$ contains frequency components at $\omega = 0$ (DC), 2π (rad/s), + 3π (rad/s).

$y(t) = 2 + 2\cos(2\pi t + 90°)$ contains frequency components at $\omega = 0$ + 2π (rad/s) w/ nothing at $\omega = 3\pi$ (rad/s)

Since $Y(\omega) = X(\omega)H(\omega) \Rightarrow H(\omega) = \dfrac{Y(\omega)}{X(\omega)}$.

$\Rightarrow$ We can determine $H(\omega)$ at $\underline{\underline{\omega = 0,\ 2\pi,\ + 3\pi \text{ rad/s}}}$

b) use phasors

$$H(0) = \frac{Y(0)}{X(0)} = \frac{2}{1} = \underline{\underline{2}}$$

$$H(2\pi) = \frac{Y(2\pi)}{X(2\pi)} = \frac{2\angle 90°}{4\angle 0°} = \underline{\underline{0.5\angle 90° = 0.5e^{j\pi/2}}} = j0.5$$

$$H(3\pi) = \frac{Y(3\pi)}{X(3\pi)} = \frac{0}{8\angle -180°} = \underline{\underline{0}}$$