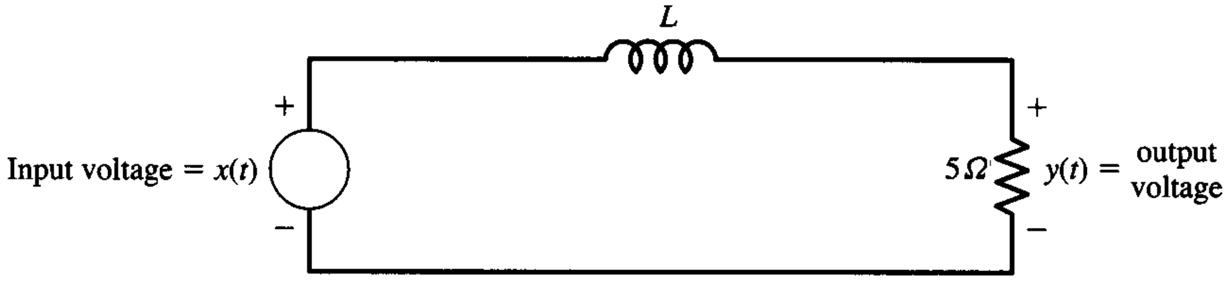
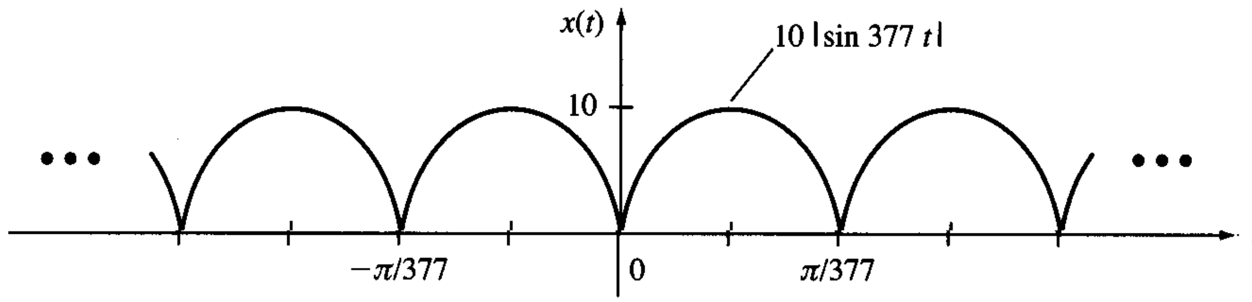
5.8 The voltage $x(t)$ shown in Figure P5.8b is applied to the RL circuit shown in Figure P5.8a.

(a) Find the value of L so that the peak of the largest ac component (harmonic) in the output response $y(t)$ is 1/30 of the dc component of the output.

(b) Plot an approximation for $y(t)$, using the truncated complex exponential Fourier series from $k = -3$ to $k = 3$.

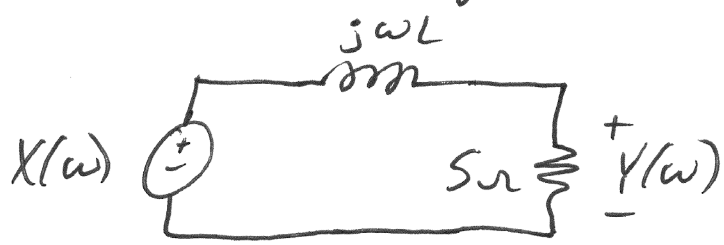


(a)



(b)

a) Put circuit in phasor form



By voltage division

$$Y(\omega) = X(\omega)\,\frac{5}{5+j\omega L}$$

$$\underline{H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{5}{5+j\omega L}}$$

Next, calculate complex exponential F.S.

per (3.20) + (3.7) $a_0 = c_0 = \frac{1}{T}\int_0^T x(t)\,dt$

where $T = \pi/377$ s + $\omega_0 = \frac{2\pi}{T} = 754$ rad/s

$x(t)$ is even $\Rightarrow b_k = 0$ + $c_k = \frac{1}{2}a_k = c_{-k}$ (3.20)

a) cont. $$C_0 = \frac{377}{\pi}\int_0^{\pi/377} 10\sin(377t)\,dt$$

$$= \frac{377}{\pi}\,10\,\frac{-\cos(377t)}{377}\Bigg|_0^{\pi/377}$$

$$\underline{C_0 = \frac{10}{\pi}\left[-\cos(\pi)+\cos(0)\right] = \frac{20}{\pi}}$$

(3.21) $$C_k = \frac{1}{T}\int_0^T x(t)\,e^{-jk\omega_0 t}\,dt$$

$$= \frac{377}{\pi}\int_0^{\pi/377} 10\sin(377t)\,e^{-jk754t}\,dt$$

$$= \frac{377(10)}{\pi}\,\frac{e^{-jk754t}\left[-jk754\sin(377t)-377\cos(377t)\right]}{(-jk754)^2+377^2}\Bigg|_0^{\pi/377}$$

$$= \frac{377(10)}{\pi}\left[\frac{e^{-jk2\pi}\left(-jk754\overset{0}{\sin(\pi)}-377\overset{-1}{\cos(\pi)}\right)}{-k^2754^2+377^2}\right.$$

$$\left. - \frac{e^0\left(-jk754\sin(0)-377\overset{1}{\cos(0)}\right)}{-k^2754^2+377^2}\right]$$

$$= \frac{377(10)}{\pi}\left[\frac{1(0+377)-1(0-377)}{377^2-2^2377^2k^2}\right]$$

$$\underline{C_k = \frac{20}{\pi(1-4k^2)} \qquad k = 0, \pm1, \pm2, \ldots}$$

$k=1$ (1st harmonic) is largest. Since $a_k = 2C_k$,

We want $2|C_1^y| = \frac{1}{30}C_0^y$

$$C_0^y = C_0 H(0) = \frac{20}{\pi}\,\frac{5}{5+j0} = \frac{20}{\pi}$$

Now, $|C_1^y| = |C_1||H(\omega_0)| = \left|\frac{20}{\pi(1-4(1)^2)}\right|\left|\frac{5}{5+j754L}\right|$

$$= 2.122066\left|\frac{5}{5+j754L}\right|$$

So, $2|C_1^y| = 4.24413\left|\frac{5}{5+j754L}\right| = \frac{1}{30}C_0^y = \frac{20}{30\pi}$

$$\hookrightarrow \frac{1}{|5+j754L|} = 0.01 \Rightarrow 5^2 + 754^2L^2 = 100^2$$

$$\Rightarrow \underline{\underline{L = 0.13246\,H}}$$

b) Per notes -

$$y(t) = \sum_{k=-\infty}^{\infty} C_k^y e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} C_k H(k\omega_0) e^{jk\omega_0 t} \quad -\infty < t < \infty$$

$$\underline{y(t) \simeq \sum_{k=-3}^{3} \frac{20}{\pi(1-4k^2)} \frac{5}{5+jk754(0.13246)} e^{jk754t} \quad -\infty < t < \infty}$$

↑ Implement using Matlab

```
% Chapter 5 problem 5.8b (chap5_5_08b.m)
% EE 313 Signals and Systems, your name, mm/dd/yyyy
% For x(t) = 10|sin(377t)|, plot the approximate output y(t)
% to input x(t) for an RL circuit with frequency response
% H(w) = 5/(5+jw) using complex exp. FS for k = -3 to 3.
clear; clc; close all;
N = 3; w0 = 754; % Define number of harmonics & fundamental freq (rad/s)
t = -0.02 : 0.0002 : 0.02; % define time vector
x = 10*abs(sin(377*t));
yN = zeros(1,length(t)); % initialize y(t) vector
for k1 = 1:1:2*N+1,
    ktmp = k1-(N+1); wk(k1) = ktmp*w0;
    H(k1) = 5./(5 + j*wk(k1)*0.13246); % RL circuit response at harmonics
    cxk(k1) = 20/pi/(1 - 4*ktmp^2); % Calculate x(t) line spectra
    cyk(k1) = H(k1)*cxk(k1); % Calculate y(t) line spectra
    yN = yN + cyk(k1).*exp(j*ktmp*w0*t); % complex exp. FS
end
yN = real(yN); % get rid of any rounding error imaginary components
plot(t,x,'b'), axis([-0.02 0.02 0 10]),
xlabel('{\itt} (sec)','fontsize',16,'fontname','times'),
ylabel('{\itx}({\itt})','fontsize',16,'fontname','times'),
title('Input {\itx}({\itt}) = 10 |sin(377{\itt})|',...
```

```
    'fontsize',16,'fontname','times'),
figure, plot(t,yN,'b'), axis([-0.02 0.02 0 10]),
xlabel('{\itt} (sec)','fontsize',16,'fontname','times'),
ylabel('{\ity}({\itt})','fontsize',16,'fontname','times'),
title('Complex exponential FS of {\ity}({\itt}) for {\itN} = 3',...
    'fontsize',16,'fontname','times'),
set(findobj('type','line'),'linewidth',2,'markersize',20)
set(findobj('type','axes'),'linewidth',2,'fontsize',12,'fontname','times')
set(findobj('type','text'),'fontsize',12,'fontname','times')
```