

5.4. Consider the system with the frequency response given by

$$H(\omega) = \frac{10}{j\omega + 10}$$

- (a) Give the output to $x(t) = 2 + 2 \cos(50t + \pi/2)$.
 (b) Sketch $|H(\omega)|$. What is the bandwidth of the filter?

➤ For part b, use Matlab to plot $|H|$ for $-50 < \omega < 50$ rad/s. Find half power bandwidth B_2 and tenth power bandwidth B_{10} .

a) Since $x(t)$ has components @ $\omega=0$ (DC) and $\omega=50$ rad/s, evaluate $H(\omega)$ at those frequencies.

$$H(0) = \frac{10}{0+10} = 1 = 1 \angle 0^\circ$$

$$H(50) = \frac{10}{j50+10} = 0.196116 \angle -78.69^\circ$$

Using linearity & (5.11),

$$\begin{aligned} y(t) &= 2 H(0) + 2 |H(50)| \cos(50t + \overset{90^\circ}{\uparrow} \frac{\pi}{2} + \angle H(50)) \\ &= 2(1) + 2(0.196116) \cos(50t + 90^\circ - 78.69^\circ) \end{aligned}$$

$$\begin{aligned} y(t) &= 2 + 0.3922 \cos(50t + 11.31^\circ) \\ &= 2 + 0.3922 \cos(50t + 0.1974) \end{aligned} \quad \left. \begin{array}{l} \text{OR} \\ \text{convert} \\ \text{to} \\ \text{radians} \end{array} \right\}$$

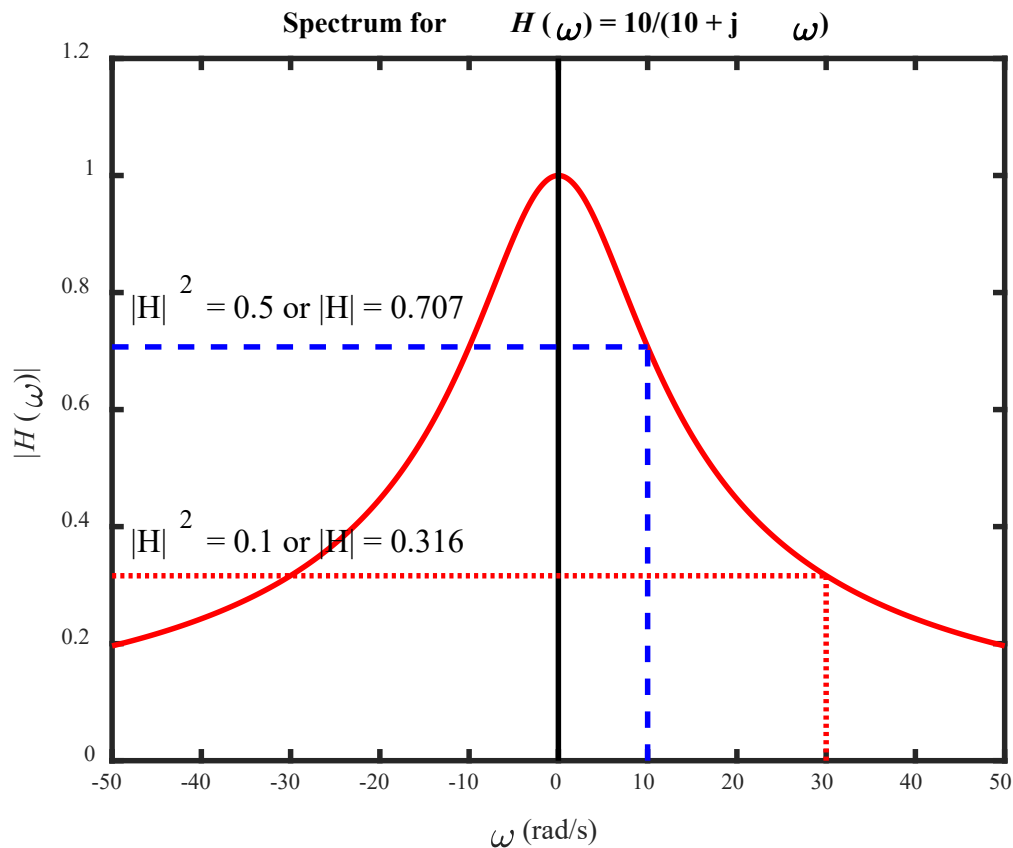
b) Power through a filter is proportional to $|H(\omega)|^2$.

$$\text{The half-power point will occur at } |H(B_2 = \omega_2)|^2 = \frac{10^2}{\omega_2^2 + 10^2} = 0.5 \Rightarrow \underline{B_2 = 10 \text{ rad/s.}}$$

$$\text{The tenth-power point will occur at } |H(B_{10} = \omega_{10})|^2 = \frac{10^2}{\omega_{10}^2 + 10^2} = 0.1 \Rightarrow \underline{B_{10} = 30 \text{ rad/s.}}$$

b) cont.

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% Chapter 5 problem 5.4b (chap5_5_04b.m)
% Plot magnitude spectrum of
% H(w) = 10/(jw+10)
clear; clc; close all;
w = -50:1/10:50; % Define frequency vector
H = 10./(10+j*w);
Hmag = abs(H); % spectrum
% Plot amplitude spectrum
plot(w,Hmag,'r-',[0 0],[0 1.2],'k-],[-50 10],[0.7071 0.7071],'b--',...
     [10 10],[0 0.7071],'b--',[-50 30],[0.316 0.316],'r:',[30 30],...
     [0 0.316],'r:'),
axis([-50 50 0 1.2]),
xlabel('\omega (rad/s)','fontsize',16,'fontname','times'),
ylabel('|H(\omega)|','fontsize',16,'fontname','times'),
title({'Spectrum for H(\omega) = 10/(10 + j\omega)'},...
      'fontsize',16,'fontname','times'),
text(-48,0.78,'|H|^2 = 0.5 or |H| = 0.707','fontsize',16,'fontname','times');
text(-48,0.38,'|H|^2 = 0.1 or |H| = 0.316','fontsize',16,'fontname','times');
set(findobj('type','line'),'linewidth',2)
set(findobj('type','axes'),'linewidth',2)
set(findobj('type','text'),'fontname','times','fontsize',14)
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➤ As shown, the half-power BW $B_2 = 10$ rad/s and tenth-power BW is $B_{10} = 30$ rad/s.