5.2 A linear time-invariant continuous-time system has the frequency response function

$$H(\omega) = \begin{cases} 2\exp(-|6-|\omega||)\exp(-j3\omega), & 4 \le |\omega| \le 12 \\ 0, & \text{all other } \omega \end{cases}$$

(a) Plot the magnitude and phase functions for $H(\omega)$.

(b) Compute and plot the output response $y(t)$ resulting from the input $x(t)$ defined in Figure P5.1.

(c) Plot the amplitude and phase spectra of $x(t)$ and $y(t)$ for $k = 0, \pm1, \pm2, \pm3, \pm4, \pm5, \pm6$.

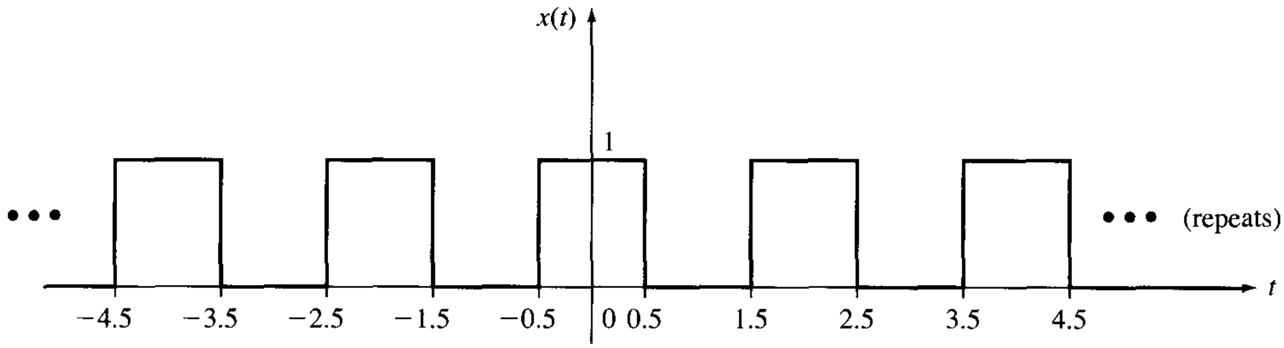
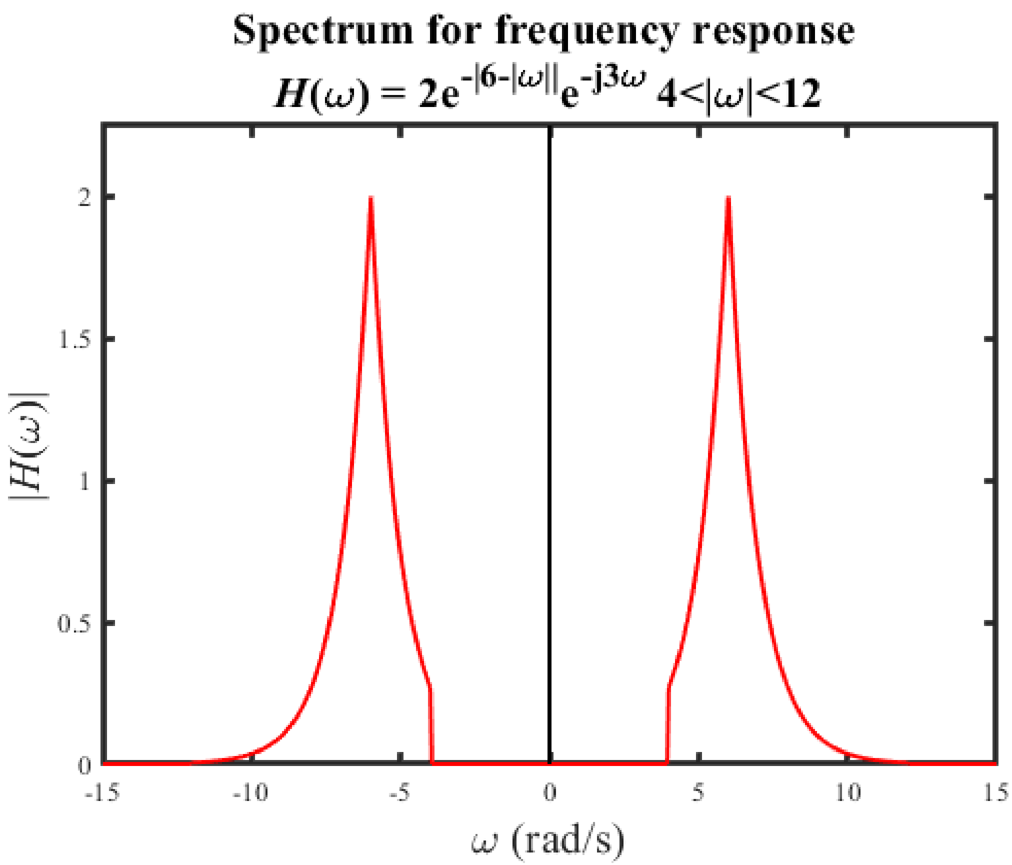


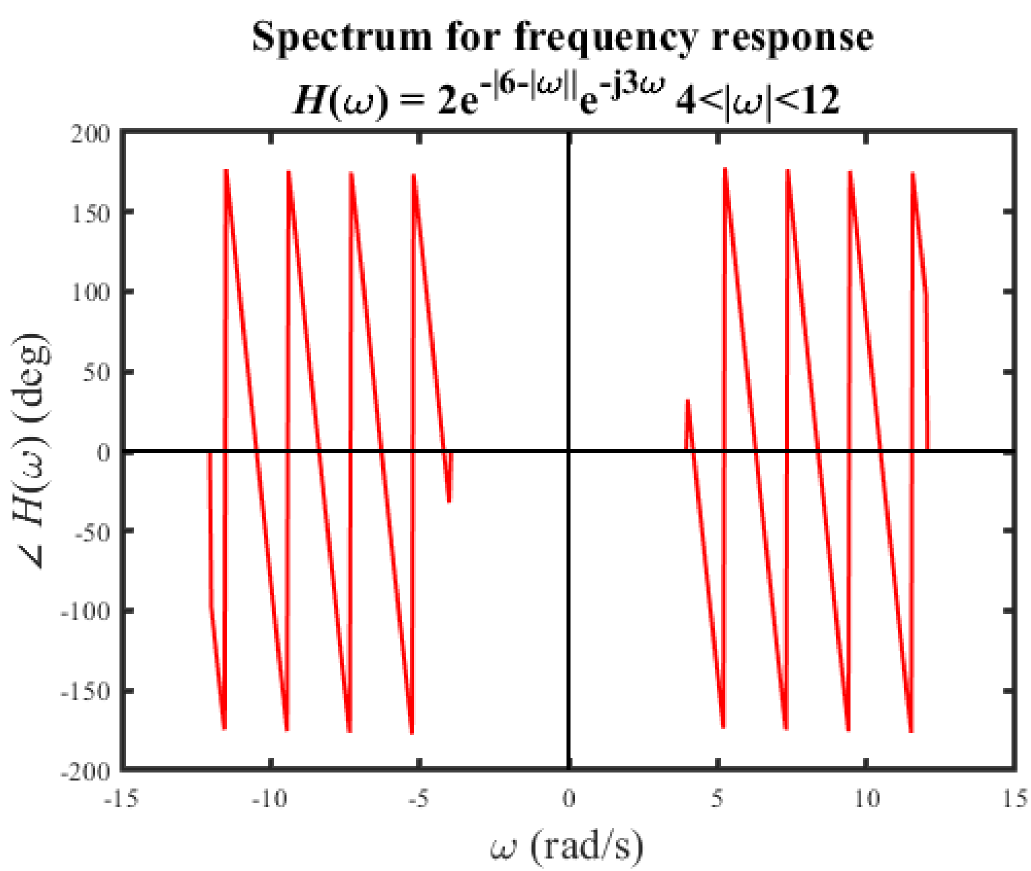
FIGURE P5.1

a)

```
% Chapter 5 problem 5.2a (chap5_5_02a.m)
% Plot magnitude and phase spectrum of
% H(w) = 2exp(-|6-|w||)exp(-j3w) 4<|w|<12
%       = 0 elsewhere
clear; clc; close all;
w = -15 : 0.05 : 15;  % Define frequency vector (rad/s)
H = 2*exp(-abs(6-abs(w))).*exp(-j*3*w);
for k = 1:length(w),
    if(abs(w(k))<4|abs(w(k))>12),
        H(k) = 0;
    end
end
Hmag = abs(H); Hang = angle(H)*180/pi; % spectrum
% Plot amplitude and phase spectrum
plot(w,Hmag,'r-',[0 0],[0 2.25],'k-'), axis([-15 15 0 2.25]),
xlabel('\omega (rad/s)','fontsize',16,'fontname','times'),
ylabel('|{\itH}(\omega)|','fontsize',16,'fontname','times'),
title({'Spectrum for frequency response ',...
    '{\itH}(\omega) = 2e^{-|6-|\omega||}e^{-j3\omega} 4<|\omega|<12'},...
    'fontsize',16,'fontname','times'),
figure, plot(w,Hang,'r-',[-15 15],[0 0],'k-',[0 0],[-200 200],'k-'),
axis([-15 15 -200 200]),
xlabel('\omega (rad/s)','fontsize',16,'fontname','times'),
ylabel('\angle {\itH}(\omega) (deg)','fontsize',16,'fontname','times'),
title({'Spectrum for frequency response ',...
    '{\itH}(\omega) = 2e^{-|6-|\omega||}e^{-j3\omega} 4<|\omega|<12'},...
    'fontsize',16,'fontname','times'),
set(findobj('type','line'),'linewidth',1.5)
set(findobj('type','axes'),'linewidth',2,'fontname','times')
```

a) cont.


Spectrum for frequency response
H(ω) = 2e^{-|6-|ω||}e^{-j3ω} 4<|ω|<12
|H(ω)|
2
1.5
1
0.5
0
-15
-10
-5
0
5
10
15
ω (rad/s)


Spectrum for frequency response
H(ω) = 2e^{-|6-|ω||}e^{-j3ω} 4<|ω|<12
∠ H(ω) (deg)
200
150
100
50
0
-50
-100
-150
-200
-15
-10
-5
0
5
10
15
ω (rad/s)

b)

From example 3.2, $x(t)$ has period $T = 2s$ and $\omega_0 = \pi$ rad/s.

→ The trig. Fourier series coefficients are

$$a_0 = 1/2, \quad a_k = \frac{2}{\pi k}\sin\left(\frac{k\pi}{2}\right) \quad k = 1, 2, 3, \ldots, \quad b_k = 0$$

and

$$x(t) = 1/2 + \frac{2}{\pi}\sum_{k=1}^{\infty} \frac{1}{k}\sin(k\pi/2)\cos(k\pi t) \quad -\infty < t < \infty$$

Which harmonics will pass through system?

$DC \to \omega_k = 0 \qquad H(0) = 0$

$k = 1 \to \omega_k = \pi \qquad H(\pi) = 0$

$k = 2 \to \omega_k = 2\pi \qquad H(2\pi) \neq 0$ However $a_2 = 0$

$k = 3 \to \omega_k = 3\pi \qquad H(3\pi) \neq 0 \longleftarrow a_3 = -\frac{2}{3\pi}$

$k \geq 4 \qquad \omega_k \geq 4\pi \quad H(\omega_k) = 0$

By (5.11), $y(t) = |H(\omega_0)|\, A\cos(\omega_0 t + \theta + \angle H(\omega_0))$

Here $A_3 = |a_3| = \left|\frac{2}{\pi(3)}\sin(3\pi/2)\right| = 0.2122$

$\theta_3 = \pi$ (Since $a_k < 0$ & $b_k = 0$)

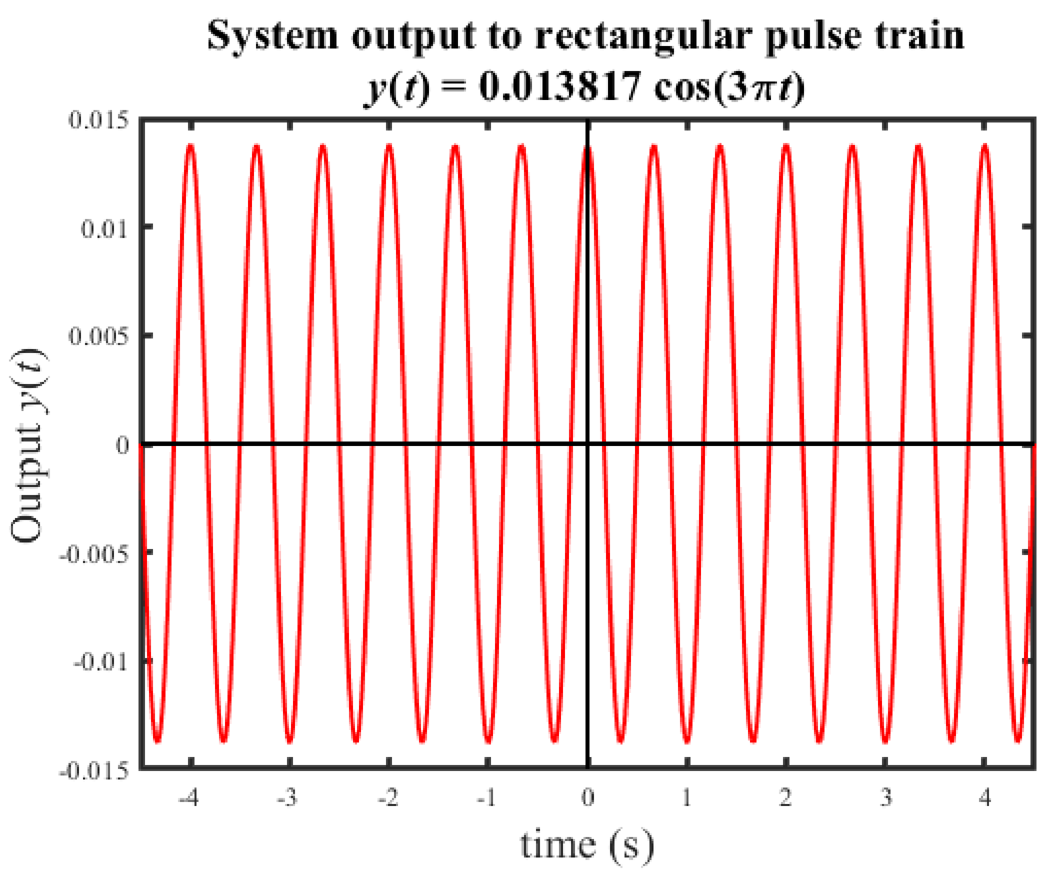
$$H(3\pi) = 2e^{-|6-3\pi|}e^{-j3(3\pi)} = 0.065113 \angle \pm\pi$$

$$y(t) = (0.065113)(0.2122)\cos(3\pi t + \pi - \pi)$$

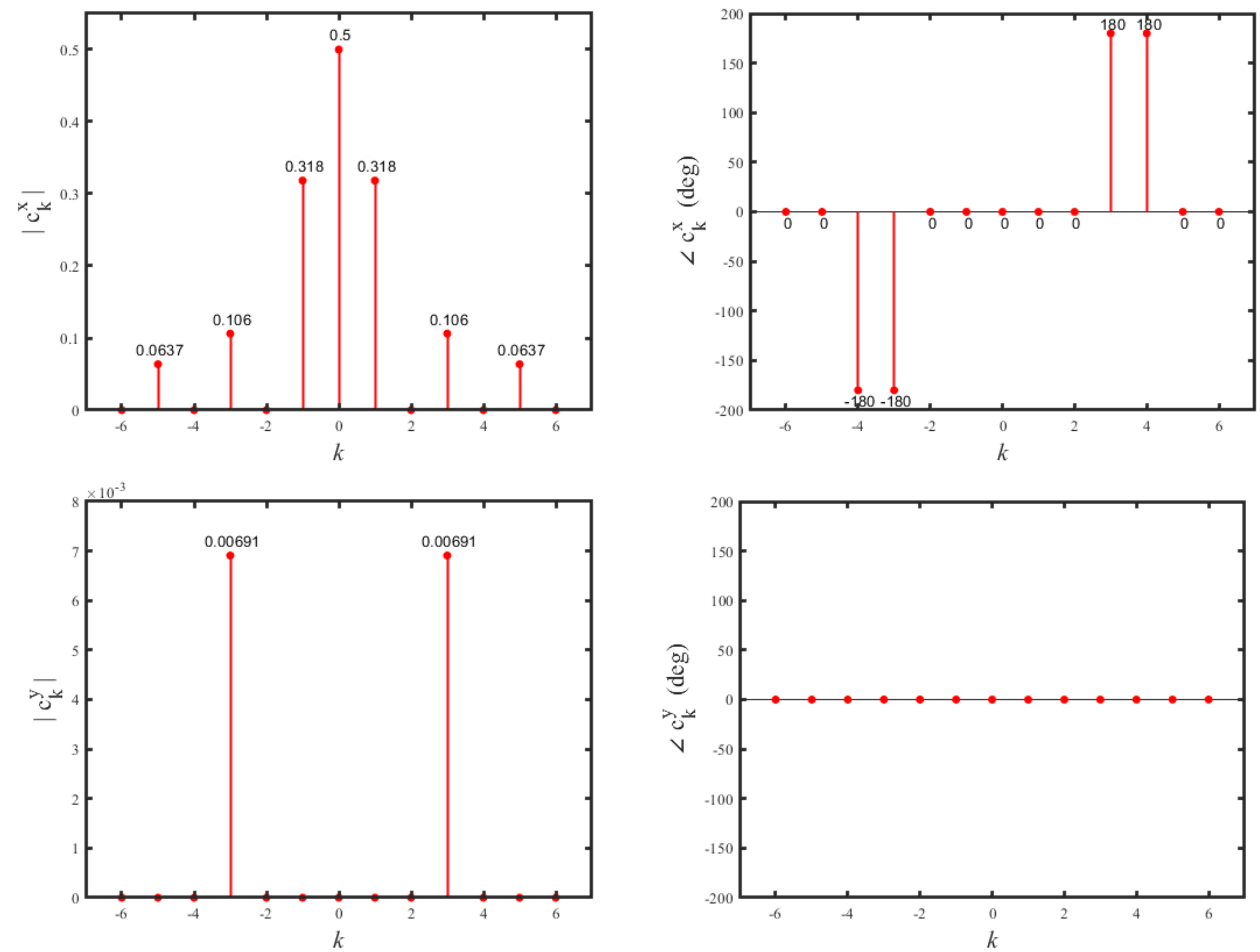
$$\underline{\underline{y(t) = 0.01382\cos(3\pi t) \quad -\infty < t < \infty}}$$

b) cont.

```
% Chapter 5 problem 5.2b (chap5_5_02b.m)
% Plot the output response y(t) to the input of the
% pulse train x(t) into the system with
% H(w) = 2exp(-|6-|w||)exp(-j3w) 4<|w|<12
%      = 0 elsewhere
clear; clc; close all;
t = -4.5:0.025:4.5; % define time vector
w3 = 3*pi;  % third harmonic frequency
% 3rd harmonic trig. FS coefficient (b3 = 0)
a3 = (2/pi/3)*sin(3*pi/2);
A3 = abs(a3); Theta3 = angle(a3); % cosine w/ phase coeff.
% freq. resp. at 3rd harmonic
H3 = 2*exp(-abs(6-abs(w3))).*exp(-j*3*w3);
H3mag = abs(H3); H3ang = angle(H3);
y = A3*H3mag*cos(w3*t + Theta3 + H3ang);
% Plot output y(t)
plot(t,y,'r-',[-4.5 4.5],[0 0],'k-',[0 0],[-0.015 0.015],'k-'),
axis([-4.5 4.5 -0.015 0.015]),
xlabel('time (s)','fontsize',16,'fontname','times'),
ylabel('Output {\ity}({\itt})','fontsize',16,'fontname','times'),
title({'System output to rectangular pulse train';...
    ['{\ity}({\itt}) = ',num2str(A3*H3mag),' cos(3\pi{\itt})']},...
    'fontsize',16,'fontname','times'),
set(findobj('type','line'),'linewidth',1.5)
set(findobj('type','axes'),'linewidth',2,'fontname','times')
```



c)




```
% Chapter 5 problem 5.2c (chap5_5_02c.m)
% Plot the phase and amplitude spectra for the pulse train
% input x(t) and output y(t) w/ system frequency response
% H(w) = 2exp(-|6-|w||)exp(-j3w) 4<|w|<12
%       = 0 elsewhere
clear; clc; close all;
N = 6; w0 = pi;
for k1 = 1:1:2*N+1,
    ktmp = k1-(N+1); wk(k1) = ktmp*pi;
    H(k1) = 2*exp(-abs(6-abs(wk(k1)))).*exp(-j*3*wk(k1));
    if(abs(wk(k1))<4|abs(wk(k1))>12),
        H(k1) = 0;
    end
    if(ktmp == 0),
        cxk(k1) = 0.5;
    else
        cxk(k1) = (1/ktmp/w0).*sin(0.5*ktmp*w0);
    end
    cyk(k1) = H(k1).*cxk(k1); % Calculate y(t) line spectra
end
k = -N:1:N; cxkmag = abs(cxk); cykmag = abs(cyk);
for k2 = 1:1:2*N+1,
    ktmp2 = k2-(N+1);
```

```
    if(ktmp2<0),
        cxkang(k2)=-angle(cxk(k2))*180/pi; cykang(k2)=-
angle(cyk(k2))*180/pi;
    else
        cxkang(k2)=angle(cxk(k2))*180/pi;
cykang(k2)=angle(cyk(k2))*180/pi;
    end
end
stem(k,cxkmag,'r.','linewidth',1.5,'markersize',18),
axis([-(N+1) N+1 0 0.55]),
xlabel('\itk','fontsize',16,'fontname','times'),
ylabel('| c^x_k |','fontsize',16,'fontname','times'),
% Label stems
for n=1:2*N+1,  %
    if(cxkmag(n)>0.0009),
        text(k(n),cxkmag(n)+0.035,['  ' num2str(cxkmag(n),3)],...
      'HorizontalAlignment','center','VerticalAlignment','top')
    end
end
figure, stem(k,cxkang,'r.','linewidth',1.5,'markersize',18),
axis([-(N+1) N+1 -200 200]),
xlabel('\itk','fontsize',16,'fontname','times'),
ylabel('\angle c^x_k  (deg)','fontsize',16,'fontname','times'),
% Label stems
for n=1:2*N+1,  %
    if(cxkang(n) > 0.1),
        text(k(n),cxkang(n)+0.45,['  ' num2str(cxkang(n),3)],...

'HorizontalAlignment','center','VerticalAlignment','bottom')
    else
        text(k(n),cxkang(n)-0.1,['  ' num2str(cxkang(n),3)],...
            'HorizontalAlignment','center','VerticalAlignment','top')
    end
end
figure, stem(k,cykmag,'r.','linewidth',1.5,'markersize',18),
axis([-(N+1) N+1 0 0.008]),
xlabel('\itk','fontsize',16,'fontname','times'),
ylabel('| c^y_k |','fontsize',16,'fontname','times'),
% Label stems
for n=1:2*N+1,  %
    if(cykmag(n)>0.0009),
        text(k(n),cykmag(n)+0.0005,['  ' num2str(cykmag(n),3)],...
      'HorizontalAlignment','center','VerticalAlignment','top')
    end
end
figure, stem(k,cykang,'r.','linewidth',1.5,'markersize',18),
axis([-(N+1) N+1 -200 200]),
xlabel('\itk','fontsize',16,'fontname','times'),
ylabel('\angle c^y_k  (deg)','fontsize',16,'fontname','times'),
set(findobj('type','line'),'linewidth',1.5,'markersize',18)
set(findobj('type','axes'),'linewidth',2,'fontname','times')
```