

Manually compute (show work) the IDFT of the DFT points  $X_k$  where  $X_0 = 5$ ,  $X_1 = 1 + j2$ ,  $X_2 = -15$ , and  $X_3 = 1 - j2$ . Plot input  $x[n]$  with each of the stems labeled.

$$\text{Use (4.40), } x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, \quad n = 0, 1, \dots, N-1 \text{ with } N = 4.$$

$$\begin{aligned} x[n] &= \frac{1}{4} \sum_{k=0}^3 X_k e^{j2\pi kn/4} = \frac{1}{4} [X_0 + X_1 e^{j2\pi(1)n/4} + X_2 e^{j2\pi(2)n/4} + X_3 e^{j2\pi(3)n/4}] \\ &= \frac{1}{4} [3 + (4 + j1)e^{j\pi n/2} - 7e^{j\pi n} + (4 - j1)e^{j3\pi n/2}], \quad n = 0, 1, 2, 3 \end{aligned}$$

$$n = 0: \quad x[0] = \frac{1}{4} [5 + (1 + j2)e^0 - 15e^0 + (1 - j2)e^0] \quad \Rightarrow \quad \underline{x[0] = -2}$$

$$n = 1: \quad x[1] = \frac{1}{4} [3 + (1 + j2)e^{j\pi(1)/2} - 15e^{j\pi(1)} + (1 - j2)e^{j3\pi(1)/2}] \quad \Rightarrow \quad \underline{x[1] = 4}$$

$$n = 2: \quad x[2] = \frac{1}{4} [5 + (1 + j2)e^{j\pi(2)/2} - 15e^{j\pi(2)} + (1 - j2)e^{j3\pi(2)/2}] \quad \Rightarrow \quad \underline{x[2] = -3}$$

$$n = 3: \quad x[3] = \frac{1}{4} [5 + (1 + j2)e^{j\pi(3)/2} - 15e^{j\pi(3)} + (1 - j2)e^{j3\pi(3)/2}] \quad \Rightarrow \quad \underline{x[3] = 6}$$

