4.5 A discrete-time signal $x[n]$ has DTFT $X(\Omega) = \dfrac{1}{e^{j\Omega} + b}$

where b is an arbitrary constant. Determine the DTFT $V(\Omega)$ of the following:

(a) $v[n] = x[n-5]$

(g) $v[n] = x^2[n]$

(h) $v[n] = x[n]e^{j2n}$

a) From Table 4.2, use time shift property

$$x[n-q] \longleftrightarrow X(\Omega)e^{-jq\Omega} \quad q \equiv \text{integer}$$

$$V(\Omega) = X(\Omega)e^{-j5\Omega}$$

$$\underline{\underline{V(\Omega) = \frac{1}{e^{j\Omega}+b}e^{-j5\Omega}}}$$

g) From Table 4.2, use the "Multiplication in the time-domain" property

$$x[n]v[n] \longleftrightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega-\lambda)V(\lambda)\,d\lambda$$

$$V(\Omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{1}{e^{j(\Omega-\lambda)}+b}\right)\left(\frac{1}{e^{j\lambda}+b}\right)d\lambda$$

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$$\underline{\underline{V(\Omega) = \frac{1}{e^{j\Omega}-b^2} \quad -\infty<\Omega<\infty}}$$

Alt. approach, $X(\Omega) = \dfrac{1}{e^{j\Omega}+b} = \left(\dfrac{1}{1+be^{-j\Omega}}\right)e^{-j\Omega}$

From Table 4.1, $a^n u[n] \longleftrightarrow \dfrac{1}{1-ae^{-j\Omega}}$ where $a=-b$

From Table 4.2, $x[n-q] \longleftrightarrow X(\Omega)e^{-jq\Omega}$ where $q=1$

So, $x[n] = (-b)^{n-1}u[n-1]$ and $x^2[n] = (-b)^{n-1}u[n-1]\,(-b)^{n-1}u[n-1]$

$$= (b^2)^{n-1}u[n-1]$$

using the above w/ $a=b^2$ + $q=1$

$$\underline{V(\Omega) = \frac{1}{1-b^2e^{-j\Omega}}\left(e^{-j\Omega}\right) = \frac{1}{e^{+j\Omega}-b^2} \quad -\infty<\Omega<\infty}$$

h) For $v[n] = x[n]\,e^{j2n}$, use the Table 4.2 property 'Multiplication by a complex exponential'

$$x[n]e^{jn\Omega_0} \leftrightarrow X(\Omega - \Omega_0) \quad w/\ \Omega_0 = 2$$

$$V(\Omega) = \left.\frac{1}{e^{j\Omega} + b}\right|_{\Omega = \Omega - 2}$$

$$\underline{\underline{V(\Omega) = \frac{1}{e^{j(\Omega - 2)} + b}}} \quad -\infty < \Omega < \infty$$