

4.4. Compute the DTFT of the following discrete-time signals; plot the amplitude and the phase spectrum for each signal:

(b) $x[n] = (0.5)^n \cos 4n u[n]$

From Table 4.1, use the DTFT pair

$$a^n u[n] \leftrightarrow \frac{1}{1 - ae^{-j\omega}} \quad |a| < 1$$

and, from Table 4.2, use the DTFT property multiply by $\cos \omega_0 n$

$$x[n] \cos(\omega_0 n) \leftrightarrow \frac{1}{2} [X(\omega + \omega_0) + X(\omega - \omega_0)]$$

where $a = 0.5$ and $\omega_0 = 4 \text{ rad}$

$$X(\omega) = \frac{1}{2} \left[\left. \frac{1}{1 - 0.5e^{-j\omega}} \right|_{\omega = \omega + 4} + \left. \frac{1}{1 - 0.5e^{-j\omega}} \right|_{\omega = \omega - 4} \right]$$

$$X(\omega) = \frac{1}{2} \left[\frac{1}{1 - 0.5e^{-j(\omega+4)}} + \frac{1}{1 - 0.5e^{-j(\omega-4)}} \right]$$

or

$$X(\omega) = \frac{1}{2} \left[\frac{e^{j(\omega+4)}}{e^{j(\omega+4)} - 0.5} + \frac{e^{j(\omega-4)}}{e^{j(\omega-4)} - 0.5} \right]$$

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% Chapter 4 problem 4.4b (chap4_4_04b.m)
% Plot DTFT of  $x[n] = (0.5)^n \cos(4n) u[n]$ 
%
clear; clc; close all;
Omega = -pi:pi/50:pi; % Define frequency vector
X1 = 0.5./(1 - 0.5*exp(-j*(Omega + 4)));
X2 = 0.5./(1 - 0.5*exp(-j*(Omega - 4)));
X = X1 + X2;
Xmag = abs(X); Xang = angle(X); % spectrum
% Plot amplitude and phase spectrum
plot(Omega,Xmag,'r-',[0 0],[0 1.5],'k-'), axis([-pi pi 0 1.5]),
xlabel('\Omega (rad)','fontsize',16,'fontname','times'),
ylabel('|X(\Omega)|','fontsize',16,'fontname','times'),
title('DTFT spectrum for  $x[n] = (0.5)^n \cos(4n) u[n]$ ',...
      'fontsize',16,'fontname','times'),
figure,
plot(Omega,Xang,'r-],[-pi pi],[0 0],'k-',[0 0],[-30 30],'k-'),
axis([-pi pi -0.5 0.5]),
xlabel('\Omega (rad)','fontsize',16,'fontname','times'),
ylabel('\angle X(\Omega) (rad)','fontsize',16,'fontname','times'),
title('DTFT spectrum for  $x[n] = (0.5)^n \cos(4n) u[n]$ ',...
      'fontsize',16,'fontname','times'),
set(findobj('type','line'),'linewidth',2)
set(findobj('type','axes'),'linewidth',2)
set(findobj('type','text'),'fontname','times')

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