3.24 A continuous-time signal $x(t)$ has Fourier transform

$$X(\omega) = \frac{1}{j\omega + b}$$

where b is a constant. Determine the Fourier transform $V(\omega)$ of the following signals:

(a) $v(t) = x(5t - 4)$

(c) $v(t) = x(t)e^{j2t}$

(f) $v(t) = x(t) * x(t)$

(h) $v(t) = \dfrac{1}{jt - b}$

a) $v(t) = x(5t-4) = x(5(t-4/5))$

use the F.T. properties:

time shift $\quad x(t-c) \Leftrightarrow X(\omega)e^{-j\omega c} \quad c = 4/5$

and

time scaling $\quad x(at) \Leftrightarrow \frac{1}{a}X(\omega/a) \quad a = 5$

to get

$$V(\omega) = \left[\frac{1}{5}\frac{1}{j\frac{\omega}{5}+b}\right] e^{-j\omega 4/5}$$

$$\underline{\underline{V(\omega) = \frac{1}{j\omega + 5b} e^{-j4\omega/5}}} \quad -\infty < \omega < \infty$$

c) $v(t) = x(t)e^{j2t}$

use FT property multiplication by $e^{j\omega_0 t}$

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(\omega - \omega_0) \quad \text{where } \omega_0 = 2$$

$$\underline{\underline{V(\omega) = \frac{1}{j(\omega-2)+b}}} \quad -\infty < \omega < \infty$$

f) From Table 3.1, the convolution in the time-domain property is

$$x(t) * v(t) \longleftrightarrow X(\omega)V(\omega)$$

Therefore, $V(\omega) = X(\omega)X(\omega)$

$$V(\omega) = \left(\frac{1}{j\omega+b}\right)^2 \quad -\infty < \omega < \infty$$

h) From Table 3.2, $x(t) = e^{-bt}u(t) \longleftrightarrow X(\omega) = \frac{1}{j\omega+b}$

Note that $-X(-\omega) = (-1)\frac{1}{j(-\omega)+b} = \frac{1}{j\omega-b}$

corresponds w/ $v(t) = \frac{1}{jt-b}$

Per Table 3.1, Duality property

$$X(t) \longleftrightarrow 2\pi x(-\omega)$$

Per Table 3.1, Time reversal property

$$x(-t) \longleftrightarrow X(-\omega)$$

Combining & using linearity

$$V(\omega) = -2\pi x(\omega) = -2\pi e^{-b\omega}u(\omega)$$