

3.17. Compute the Fourier transform of the following signals, using the symbolic manipulator to perform the integrations. In each case, plot the signal $x(t)$ and the magnitude $|X(\omega)|$ of the Fourier transform.

(d) $x(t) = e^{-t}(\cos 5t + \cos 30t)u(t)$

- Find $X(\omega)$ by direct integration. Using Matlab, plot $x(t)$ for $0 \leq t \leq 4$ s and $|X(\omega)|$ for $0 \leq \omega \leq 50$ rad/s.

Per (3.30), $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \quad -\infty < \omega < \infty$

$$\begin{aligned} X(\omega) &= \int_{-\infty}^{\infty} e^{-t} (\cos 5t + \cos 30t) u(t) e^{-j\omega t} dt \\ &= \int_0^{\infty} e^{(-1-j\omega)t} \cos 5t dt + \int_0^{\infty} e^{(-1-j\omega)t} \cos 30t dt \end{aligned}$$

From Schaum's Mathematical Handbook

$$\int e^{ax} \cos bx dx = \frac{e^{ax} [a \cos bx + b \sin bx]}{a^2 + b^2}$$

$$\begin{aligned} X(\omega) &= \frac{e^{(-1-j\omega)t} [(-1-j\omega) \cos 5t + 5 \sin 5t]}{(-1-j\omega)^2 + 5^2} \Big|_{t=0}^{\infty} \\ &\quad + \frac{e^{(-1-j\omega)t} [(-1-j\omega) \cos 30t + 30 \sin 30t]}{(-1-j\omega)^2 + 30^2} \Big|_{t=0}^{\infty} \end{aligned}$$

$$\begin{aligned} &= \left[0 - \frac{e^0 [(-1-j\omega) \cos 5 \cdot 0 + 5 \sin 5 \cdot 0]}{(-1-j\omega)^2 + 5^2} \right] \\ &\quad + \left[0 - \frac{e^0 [(-1-j\omega) \cos 30 \cdot 0 + 30 \sin 30 \cdot 0]}{(-1-j\omega)^2 + 30^2} \right] \end{aligned}$$

$$\underline{X(\omega) = \frac{1+j\omega}{(1+j\omega)^2 + 5^2} + \frac{1+j\omega}{(1+j\omega)^2 + 30^2} \quad -\infty < \omega < \infty}$$

As a check, let Matlab do the integrating:

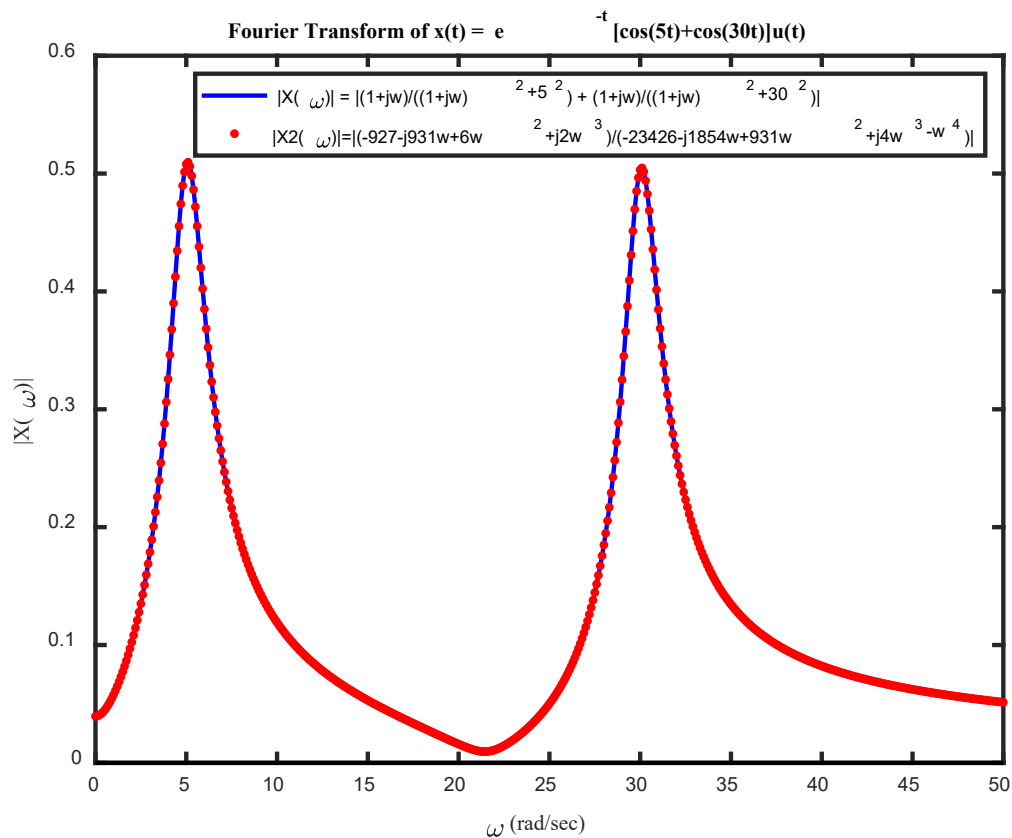
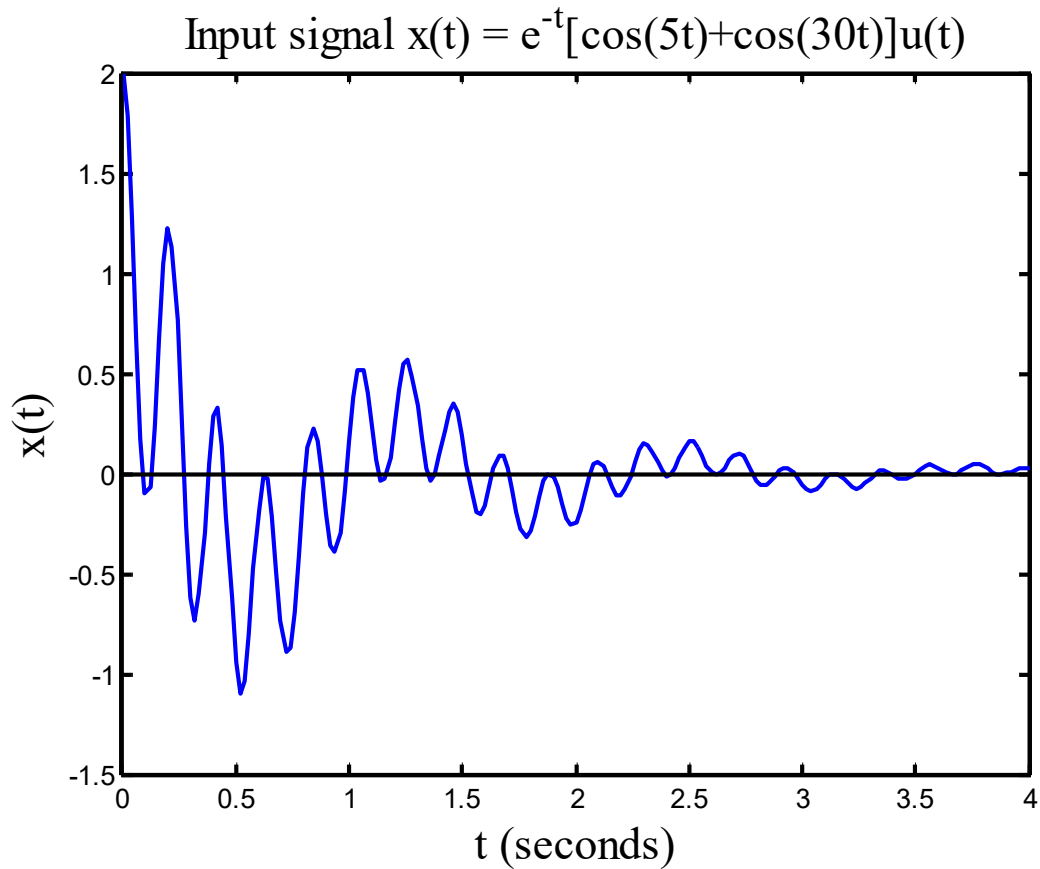
```
% Chapter 3 Fourier transform (chap3_3_17d_FT.m)
%
% Compute the Fourier transform X(w)
% of x(t) = e^(-t)*(cos(5t)+cos(30t))*u(t) .
%
clear;clc;close all;
syms x t X w % Define symbolic variables
% Compute Fourier transform using MATLAB symbolic operations
X2 = simplify(int(exp(-t)*(cos(5*t)+cos(30*t))*exp(-j*w*t),t,0,inf))
```

MATLAB Command Window output (I manually deleted blank lines)

```
X2 = (-927-931*i*w+6*w^2+2*i*w^3)/(-23426-1854*i*w+931*w^2+4*i*w^3-w^4)
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```
% Chapter 3 Fourier transform (chap3_3_17d_plot.m)
%
% Plot signal x(t) = e^(-t)*(cos(5t)+cos(30t))*u(t) .
% Then, plot the magnitude of the Fourier transform(s):
% X(w) = (1+jw)/[(1+jw)^2 + 5^2]+(1+jw)/[(1+jw)^2 + 30^2] or
% X2(w)=(-927-j931w+6w^2+j2w^3)/(-23426-j1854w+931w^2+j4w^3-w^4)
%
clear; clc; close all;
% Compute x(t) for plotting
t = 0:0.02:4;
x = exp(-t).*(cos(5*t) + cos(30*t));
plot(t,x,'b-',[0 4],[0,0],'k-'), axis([0 4 -1.5 2]),
title('Input signal x(t) = e^{-t}[cos(5t)+cos(30t)]u(t)',...
      'fontsize',16,'fontname','times')
xlabel('\itt (seconds)','fontsize',16,'fontname','times')
ylabel(['\itx'('\itt)'],'fontsize',16,'fontname','times')
% Compute X(w) for plotting
w = 0:0.1:50;
X = (1+j*w)./((1+j*w).^2 + 25) + (1+j*w)./((1+j*w).^2 + 900);
X2 = (-927-931*j*w+6*w.^2+2*j*w.^3)./...
      (-23426-1854*i*w+931*w.^2+4*j*w.^3-w.^4);
figure,
plot(w,abs(X),'b-',w,abs(X2),'r.',[0 50],[0,0],'k-'), axis([0 50 0 0.6]),
title('Fourier Transform of x(t) = e^{-t}[cos(5t)+cos(30t)]u(t)',...
      'fontsize',14,'fontname','times')
xlabel('\omega (rad/sec)','fontsize',16,'fontname','times')
ylabel('|X(\omega)|','fontsize',16,'fontname','times')
legend('|X(\omega)|=|(1+jw)/((1+jw)^2+5^2) + (1+jw)/((1+jw)^2+30^2)|',...
      '|X2(\omega)|=(-927-j931w+6w^2+j2w^3)/(-23426-j1854w+931w^2+j4w^3-w^4)|')
set(findobj('type','line'),'linewidth',2)
set(findobj('type','line'),'markersize',12)
set(findobj('type','axes'),'linewidth',2,'fontsize',12)
```



Both solutions are the same!