3.10 For each of the periodic signals shown in Figure P3.3, do the following:

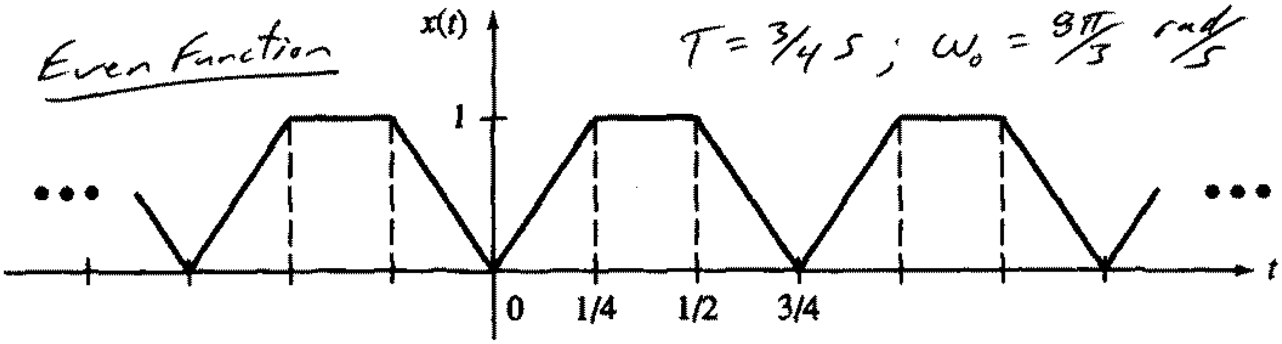
(i) Compute the complex exponential Fourier series. You may want to use the MATLAB Symbolic Math Toolbox to solve for the coefficients.

(ii) Sketch the amplitude and phase spectra for $k = \pm 1, \pm 2, \pm 3, \pm 4, \pm 5$.

(iii) Plot the truncated complex exponential series for $N = 1$, $N = 5$, and $N = 30$.

Even Function

$T = 3/4 \, s \; ; \; \omega_0 = 8\pi/3 \; rad/s$



(f)

(i) From 3.3f $-$ $a_0 = 2/3$

$$a_k = \frac{3}{\pi^2 k^2}\left[\cos\left(\frac{2\pi}{3}k\right) - 1\right] \qquad k = 1, 2, 3, \ldots$$

$$b_k = 0$$

Using (3.20), $\underline{c_0 = a_0 = 2/3 = 0.\bar{6}}$

$$c_k = 1/2\,(a_k - j\underbrace{b_k}_{\to 0}) = \frac{3}{2\pi^2 k^2}\left[\cos\left(\frac{2\pi}{3}k\right) - 1\right] = c_{-k}$$

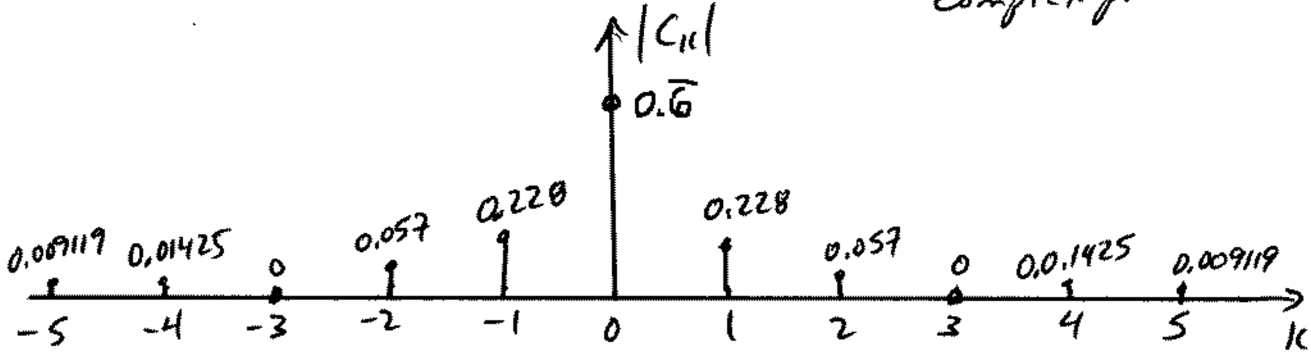
By (3.19), $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \quad -\infty < t < \infty$

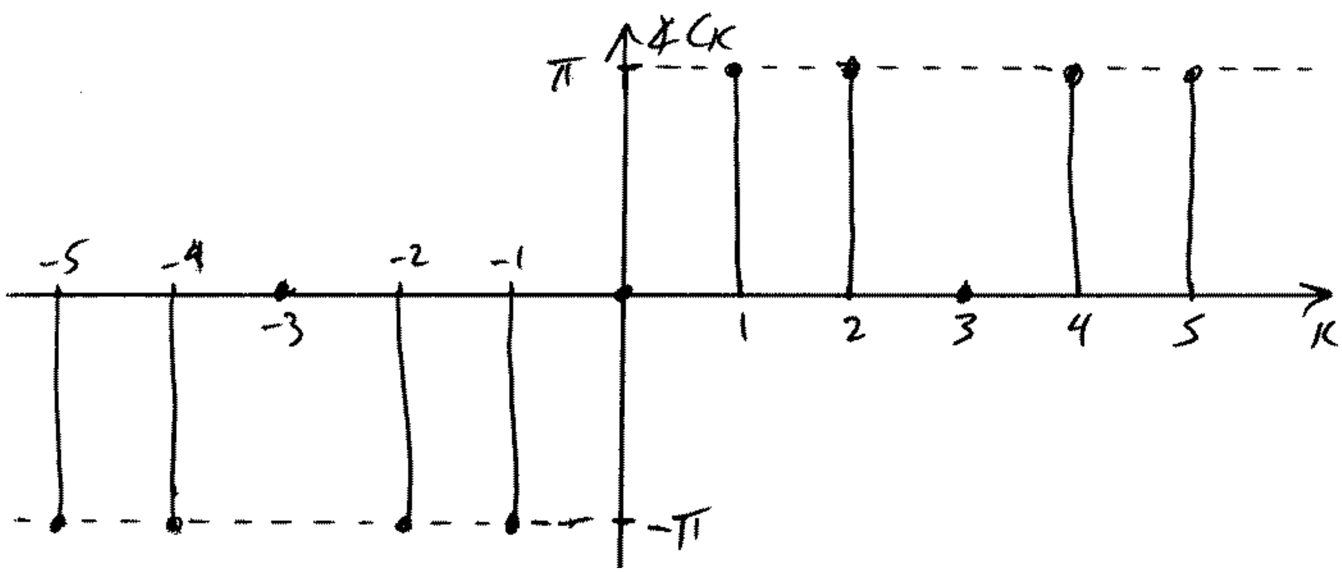
$$\boxed{x(t) = 0.\bar{6} + \sum_{\substack{k=-\infty \\ (k \neq 0)}}^{\infty} \frac{3}{2\pi^2 k^2}\left[\cos\left(\frac{2\pi}{3}k\right) - 1\right] e^{jk\frac{8\pi}{3}t} \quad -\infty < t < \infty}$$

(ii)

K	$C_K = C_{-K}$	$\lvert C_K \rvert = \lvert C_{-K} \rvert$	$\measuredangle C_K$	$\measuredangle C_{-K}$
± 1	-0.2280	0.228	$\pm\pi$	$\mp\pi$
± 2	-0.0570	0.0570	$\pm\pi$	$\mp\pi$
± 3	0	0	0	0
± 4	-0.01425	0.01425	$\pm\pi$	$\mp\pi$
± 5	-0.009119	0.009119	$\pm\pi$	$\mp\pi$

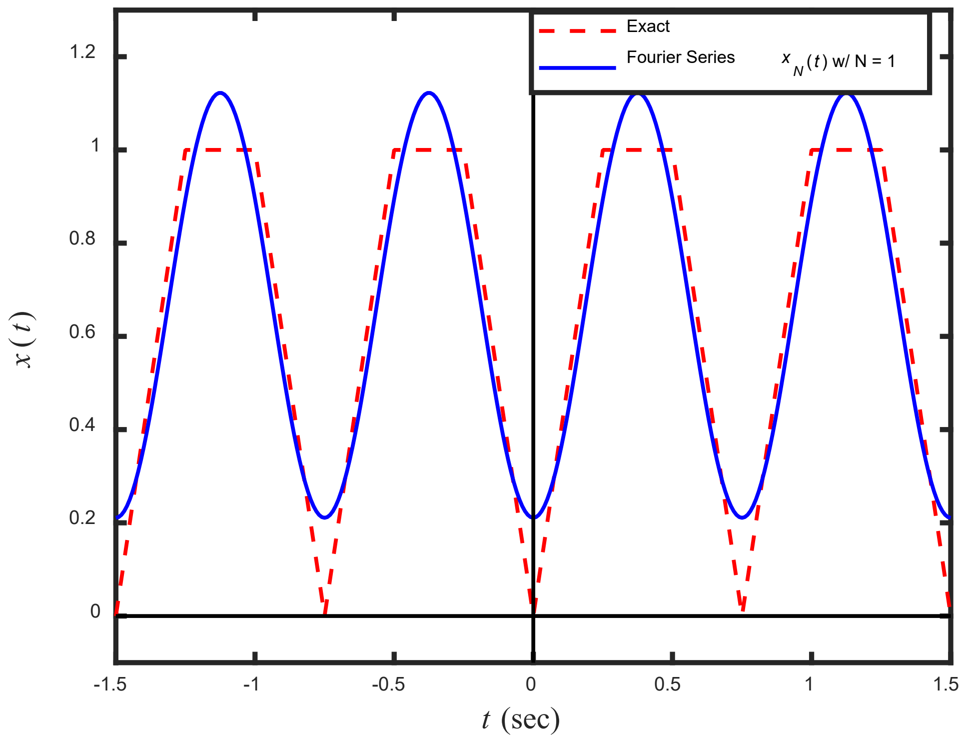
$\pm\pi$ same point on complex plane



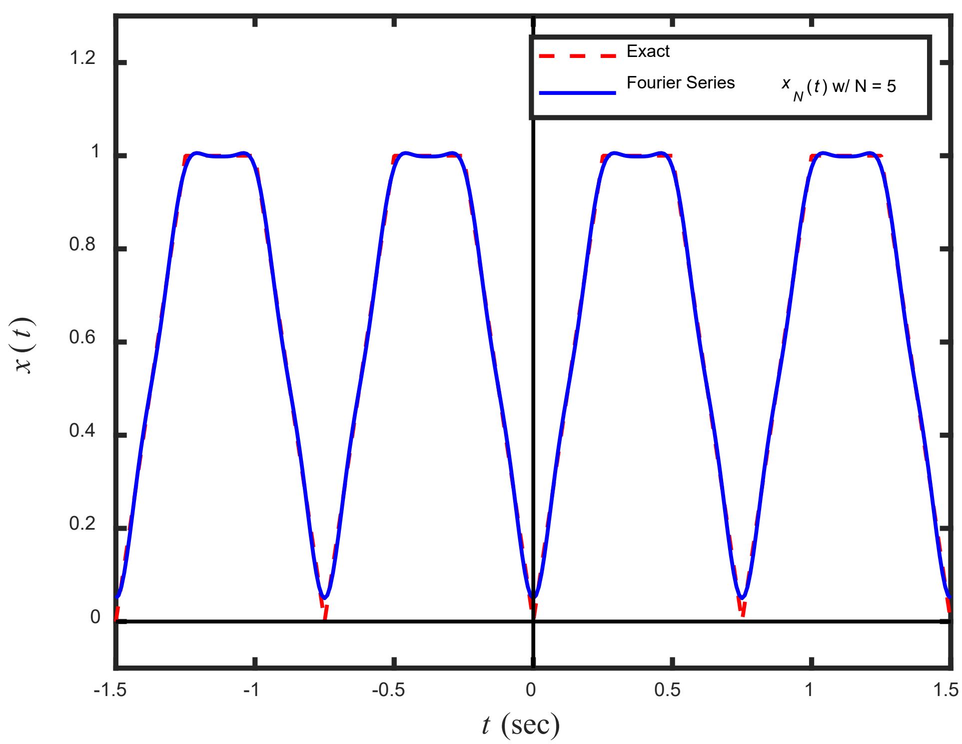


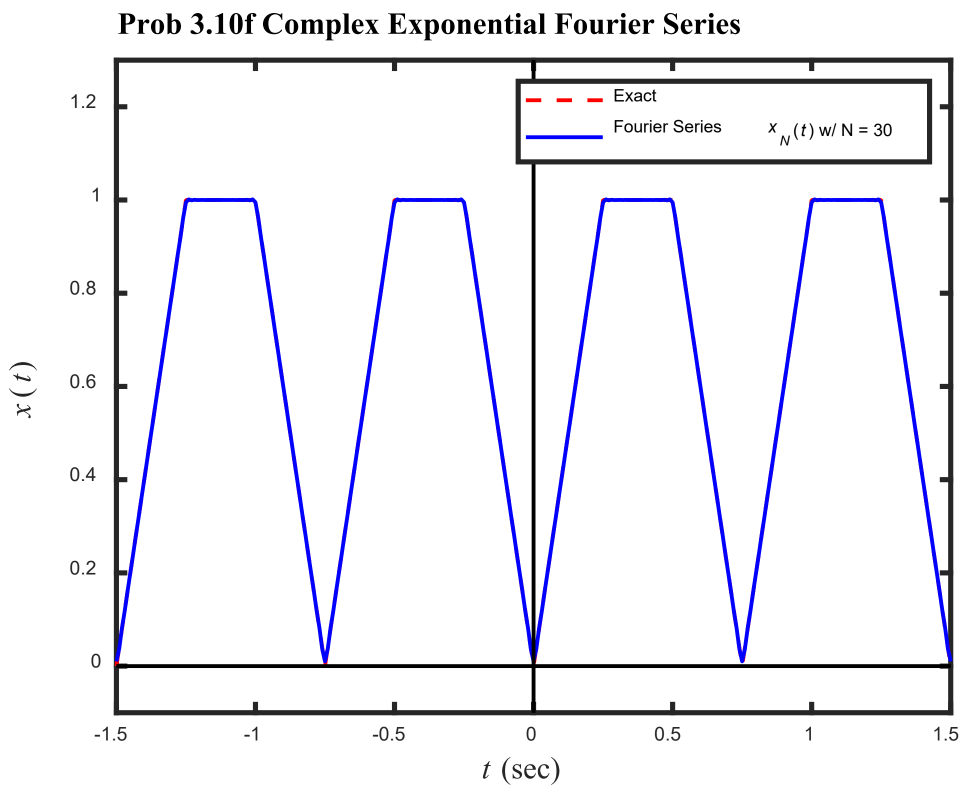
Extra credit- (iii)

Prob 3.10f Complex Exponential Fourier Series



Prob 3.10f Complex Exponential Fourier Series





Note: Since $x(t)$ does NOT have step discontinuities, the complex exponential Fourier series does NOT suffer from Gibb's phenomenon. With 30 harmonics, it is a nearly perfect representation of $x(t)$.

```
% Chapter 3 Fourier series (chap3_3_10f.m)
% Computes complex exponential Fourier series for 3.10f
clear;clc;close all;
x = [0,1,1,0,1,1,0,1,1,0,1,1,0]; % values for exact x(t)
tx = [-1.5,-1.25,-1,-0.75,-0.5,-0.25,0,0.25,0.5,0.75,1,1.25,1.5];
t = -1.5:0.01:1.5; % Compute complex exp. F.S. for this range
N = input('Number of harmonics ');
c0 = 2/3; w0 = 8*pi/3;
xN = c0*ones(1,length(t));    % dc component
for k=1:N,  % add harmonics
  cposk = (3/2/pi^2/k^2)*(cos(2*pi/3*k)-1);    % ck coeff. for k > 0
  cnegk = (3/2/pi^2/(-k)^2)*(cos(2*pi/3*-k)-1); % ck coeff. for k < 0
  xN = xN + cposk*exp(j*k*w0*t) + cnegk*exp(j*-k*w0*t);
end
plot(tx,x,'r--',t,xN,'b-',[-3 3],[0,0],'k-',[0 0],[-0.5,2.5],'k-'),
axis([-1.5 1.5 -0.1 1.3]),
title('Prob 3.10f Complex Exponential Fourier Series',...
    'fontsize',16,'fontname','times')
legend(' Exact',[' Fourier Series {\itx_N}({\itt}) w/ N = ',num2str(N)])
xlabel('{\itt} (sec)','fontsize',16,'fontname','times')
ylabel(['{\itx}({\itt})'],'fontsize',16,'fontname','times')
set(findobj('type','line'),'linewidth',1.5,'markersize',18)
set(findobj('type','axes'),'linewidth',2)
```