3.4 Express each of the trigonometric Fourier series found in Problem 3.3 in cosine-with-phase form.
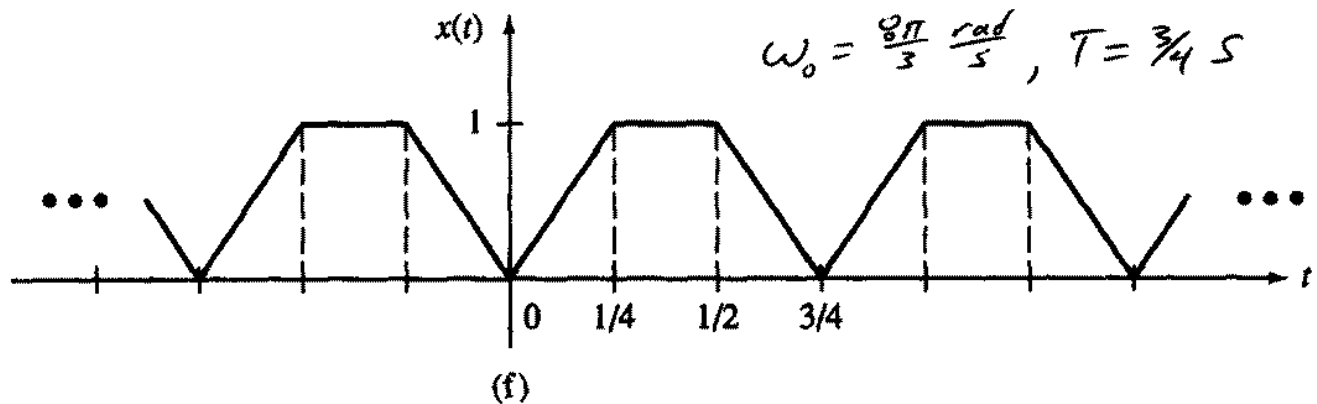

$$\omega_0 = \frac{8\pi}{3} \frac{rad}{s}, \quad T = \frac{3}{4} s$$

(f)

From 3.3f —　$a_0 = \frac{2}{3}$

$$a_k = \frac{3}{\pi^2 k^2}\left[\cos\left(\frac{2\pi}{3}k\right) - 1\right] \quad k = 1, 2, 3, \ldots$$

$$b_k = 0$$

Option 1 — let A_k be positive or negative, then

$$A_k = a_k \quad and \quad \theta_k = 0$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \theta_k)$$

← same as 3.3f

$$x(t) = \frac{2}{3} + \sum_{k=1}^{\infty}\left(\frac{3}{\pi^2 k^2}\left[\cos\left(\frac{2\pi}{3}k\right)-1\right]\right)\cos\left(k\frac{8\pi}{3}t\right) \quad -\infty < t < \infty$$

Option 2 — Be strict w/ $A_k \geq 0$

$$(3.9) \quad A_k = \sqrt{a_k^2 + b_k^2} = |a_k|$$

$$A_k = \frac{3}{\pi^2 k^2}\left|\cos\left(\frac{2\pi}{3}k\right) - 1\right| \quad k = 1, 2, 3, \ldots$$

Note:　$a_k < 0$　for all $k \neq 3, 6, 9, \ldots$

$\qquad a_k = 0$　for $k = 3, 6, 9, \ldots$

$(3,10)$ $\theta_{k} = \begin{cases} \tan^{-1}\left(\frac{-b_{k}}{a_{k}}\right) & a_{k} \geq 0 \\ \pi + \tan^{-1}\left(\frac{-b_{k}}{a_{k}}\right) & a_{k} < 0 \end{cases}$

$\rightarrow$ since $b_{k} = 0$, $\tan^{-1}(0) = 0$

$\rightarrow$ use $a_{k} = 0$ for $k = 3, 6, 9, \dots$

     & $a_{k} < 0$ for k elsewhere

$\theta_{k} = \begin{cases} 0 & k = 3, 6, 9, \dots \\ \pi & \text{all other } k \end{cases}$

Since $A_{k} \cos\left(k\omega_{0}t + \theta_{k}\right) = 0$ for $k = 3, 6, 9, \dots,$

        $\searrow 0 \text{ or } \underline{\pi}$

I will write $(3,8)$ as

$$X(t) = a_{0} + \sum_{k=1}^{\infty} A_{k} \cos\left(k\omega_{0}t + \theta_{k}\right)$$

$$X(t) = \tfrac{2}{3} + \sum_{k=1}^{\infty} \left(\frac{3}{\pi^{2}k^{2}} \left|\cos\left(\tfrac{2\pi}{3}k\right) - 1\right|\right) \cos\left(\tfrac{8\pi}{3}kt + \pi\right)$$

$$-\infty < t < \infty$$