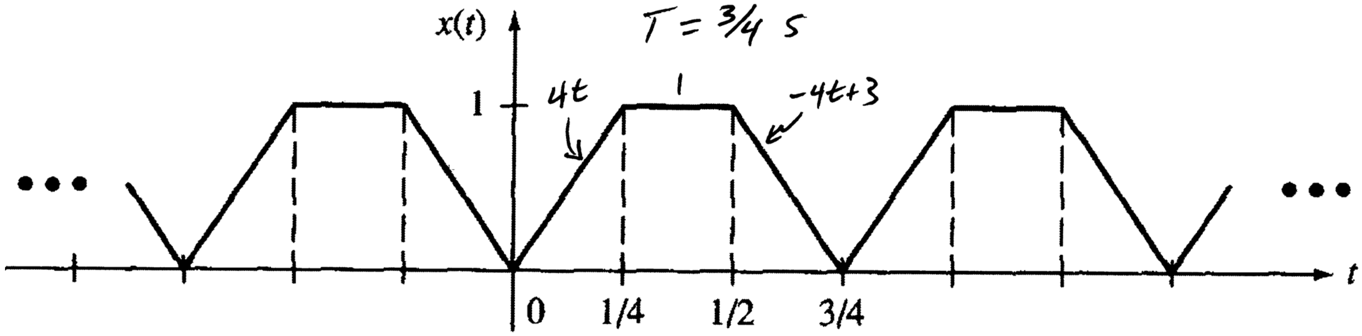
For 3.3f/3.4f, compute the exact overall time-average 'power' in $x(t)$ as well as the 'power' contained in the dc as well as each of the first three harmonics of the Fourier series $x_N(t)$. Then, find the total power in the DC + first 3 harmonics. Express answers both as a number and as a percentage of the total 'power'. Tabulate results (see below).



(f)

$$(3.28)\quad P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt = \frac{1}{T}\int_0^T x^2(t)\,dt$$

$$= \frac{1}{3/4}\int_0^{1/4}(4t)^2\,dt + \frac{1}{3/4}\int_{1/4}^{1/2} 1^2\,dt + \frac{1}{3/4}\int_{1/2}^{3/4}(-4t+3)^2\,dt$$

$$= \frac{4}{3}\,\frac{16t^3}{3}\Big|_0^{1/4} + \frac{4}{3}\,t\Big|_{1/4}^{1/2} + \frac{4}{3}\left[\frac{16t^3}{3} - \frac{24t^2}{2} + 9t\right]\Big|_{1/2}^{3/4}$$

$$= \frac{64}{9}\left[(1/4)^3 - 0\right] + \frac{4}{3}\left[1/2 - 1/4\right] + \frac{4}{3}\Big[\frac{16}{3}\left[(3/4)^3 - (1/2)^3\right]$$

$$- 12\left[(3/4)^2 - (1/2)^2\right] + 9\left[3/4 - 1/2\right]\Big]$$

$$= 0.\overline{11} + 0.\overline{33} + \frac{4}{3}\left[1.58\overline{3} - 3.75 + 2.25\right]$$

$$\underline{\underline{P = 0.\overline{55}}} \quad \text{(exact)}$$

From 3.3f, $a_0 = 2/3$, $a_k = \frac{3}{\pi^2 k^2}\left[\cos\left(\frac{2\pi}{3}k\right) - 1\right]$ $k = 1, 2, \ldots$

$b_k = 0$ (even function)

Per Parseval's Theorem (Notes from 3.27)

$$P = a_0^2 + \frac{1}{2}\sum_{k=1}^{\infty}\left(a_k^2 + b_k^2\right)$$

From this expression

$$P_{dc} = a_0^2 = (2/3)^2 = \underline{0.\overline{44}}$$

$$k=1 \quad P_{1st} = 1/2\, a_1^2 = 1/2 \left[\frac{3}{\pi^2 1^2}\left(\cos\left(\frac{2\pi}{3} 1\right) - 1\right)\right]^2 = \underline{0.103943}$$

$$k=2 \quad P_{2nd} = 1/2\, a_2^2 = 1/2 \left[\frac{3}{\pi^2 2^2}\left(\cos\left(\frac{2\pi}{3} 2\right) - 1\right)\right]^2 = \underline{0.00649644}$$

$$k=3 \quad P_{3rd} = 1/2\, a_3^2 = 1/2 \left[\frac{3}{\pi^2 3^2}\left(\cos\left(\frac{2\pi}{3} 3\right) - 1\right)\right]^2 = \underline{0}$$

$$P_{dc}/P = \frac{0.\overline{44}}{0.\overline{55}} = 0.8 \text{ or } \underline{80\%}$$

$$P_{1st}/P = \frac{0.103943}{0.\overline{55}} = 0.187097 \text{ or } \underline{18.71\%}$$

$$P_{2nd}/P = \frac{0.00649644}{0.\overline{55}} = 0.0116936 \text{ or } \underline{1.169\%}$$

$$P_{dc} + P_{1st} + P_{2nd} + P_{3rd} = 0.\overline{44} + 0.103943 + 0.00649644 + 0 = \underline{0.554884}$$

$$P_{dc+3}/P = \frac{0.554884}{0.\overline{55}} = 0.998791 \text{ or } \underline{99.879\%}$$

Term(s)	**'power'**	**% of total 'power'**
Exact	0.555555	100
DC	0.444444	80
First harmonic	0.103943	18.71
Second harmonic	0.006496	1.169
Third harmonic	0	0
DC + first 3 harmonics	0.554884	99.879