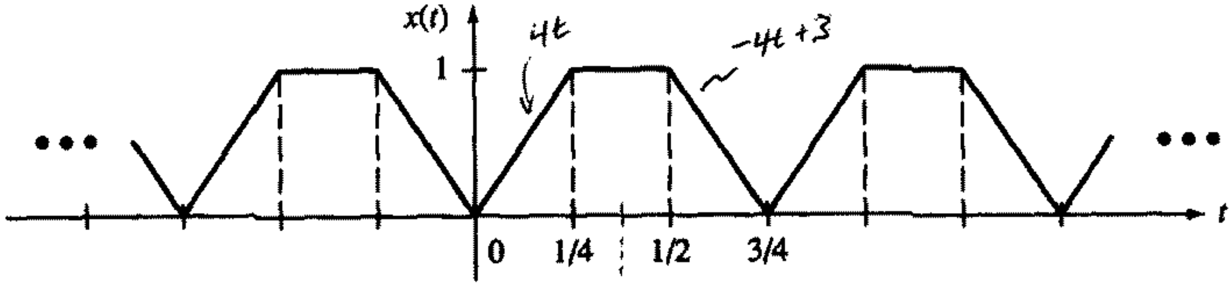
3.3. Compute the (sine/cosine) trigonometric Fourier series for each of the periodic signals shown in Figure P3.3. Use even or odd symmetry whenever possible.



(f)

$\Rightarrow$ Even function, $T = 3/4\ s$, $\omega_0 = \frac{2\pi}{T} = \frac{8\pi}{3}$ rad/s

(3.7) $a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{3/4}\int_0^{1/4} 4t\,dt + \frac{1}{3/4}\int_{1/4}^{1/2} 1\,dt + \frac{1}{3/4}\int_{1/2}^{3/4}(-4t+3)\,dt$

$= 4/3\left(4t^2/2\right)\Big|_0^{1/4} + 4/3\,(t)\Big|_{1/4}^{1/2} + 4/3\left(-\frac{4t^2}{2}+3t\right)\Big|_{1/2}^{3/4}$

$= 4/3\left[1/8 - 0\right] + 4/3\left[1/2 - 1/4\right] + 4/3\left[(-1.125+2.25)-(-0.5+1.5)\right]$

$\underline{a_0 = 0.6\overline{66} = 2/3}$

(3.5) $a_K = 2/T\int_0^T x(t)\cos(K\omega_0 t)\,dt \overset{\text{even-even}}{=} 4/T\int_0^{T/2} x(t)\cos(K\omega_0 t)\,dt$

$= \frac{4}{3/4}\int_0^{1/4} 4t\cos\left(K\frac{8\pi}{3}t\right)dt + \frac{4}{3/4}\int_{1/4}^{3/8}(1)\cos\left(K\frac{8\pi}{3}t\right)dt$

$= \frac{64}{3}\left[\frac{\cos\left(\frac{8\pi}{3}Kt\right)}{\left(\frac{8\pi}{3}K\right)^2} + \frac{t\sin\left(\frac{8\pi}{3}Kt\right)}{8\pi/3\,K}\right]\Bigg|_{t=0}^{1/4} + \frac{16}{3}\left(\frac{\sin\left(8\pi/3\,Kt\right)}{\frac{8\pi}{3}K}\right)\Bigg|_{t=1/4}^{3/8}$

$= \frac{64}{3}\left[\left(\frac{\cos\left(\frac{8\pi}{3}K\,1/4\right)}{\left(\frac{8\pi}{3}K\right)^2} + \frac{1/4\sin\left(\frac{8\pi}{3}K\,1/4\right)}{8\pi/3\,K}\right) - \left(\frac{1}{\left(\frac{8\pi}{3}K\right)^2}+0\right)\right]$

$+ \frac{16}{3}\left[\frac{\sin\left(\frac{8\pi}{3}K\,3/8\right)}{8\pi/3\,K} - \frac{\sin\left(\frac{8\pi}{3}K\,1/4\right)}{8\pi/3\,K}\right]$

$$a_k = \frac{64}{3}\left[\frac{9\cos\left(\frac{2\pi}{3}k\right)}{64\pi^2 k^2} + \cancel{\frac{3\sin\left(\frac{2\pi}{3}k\right)}{32\pi k}} - \frac{9}{64\pi^2 k^2}\right]$$

cancel

$$+\frac{16}{3}\left[\underbrace{\cancel{\frac{3\sin(\pi k)}{8\pi k}}}_{\to 0} - \cancel{\frac{3\sin\left(\frac{2\pi}{3}k\right)}{8\pi k}}\right]$$

$$= \frac{3}{\pi^2 k^2}\cos\left(\frac{2\pi}{3}k\right) - \frac{3}{\pi^2 k^2}$$

$$a_k = \frac{3}{\pi^2 k^2}\left[\cos\left(\frac{2\pi}{3}k\right) - 1\right] \quad k = 1, 2, 3, \ldots$$

Since $x(t)$ is an even function

$$b_k = 0$$

The trigonometic Fourier Series is

$$(3.4) \quad x(t) = a_0 + \sum_{k=1}^{\infty}\left[a_k\cos(k\omega_0 t) + b_k \sin(k\omega_0 t)\right]$$

$$x(t) = 2/3 + \sum_{k=1}^{\infty}\left(\frac{3}{\pi^2 k^2}\left[\cos\left(\frac{2\pi}{3}k\right) - 1\right]\right)\cos\left(k\frac{8\pi}{3}t\right) \quad -\infty < t < \infty$$