2.32. A causal linear time-invariant continuous-time system has impulse response

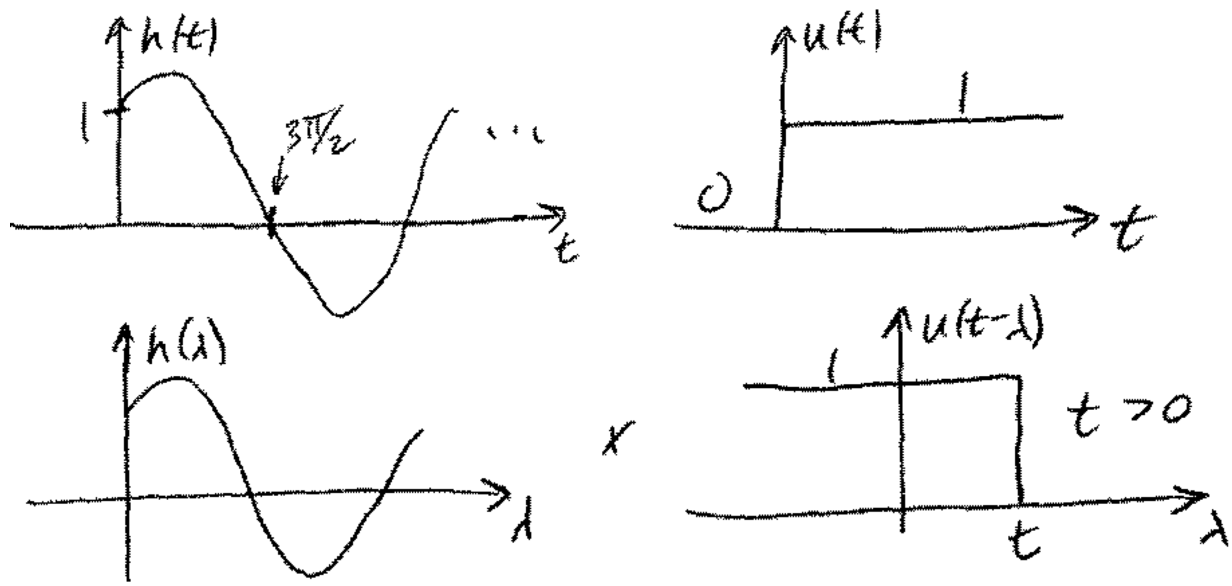
$$h(t) = e^{-t} + \sin t, t \geq 0$$

(a) Compute the output response for all $t \geq 0$ when the input is the unit-step function $u(t)$.

(b) Compute the output response $y(t)$ for all $t \geq 0$ resulting from the input $u(t) - u(t-2)$.

a)

$$g(t) = h(t) * u(t) = u(t) * h(t) = \int_0^t h(\lambda)u(t-\lambda)\,d\lambda$$



$$y(t) = \int_0^t \left(e^{-\lambda} + \sin\lambda\right)(1)\,d\lambda$$

$$y(t) = \left(-e^{-\lambda} - \cos\lambda\right)\Big|_0^t = \left(-e^{-t} - \cos t\right) - \left(-e^{0} - \cos 0\right)$$

$$\underline{g(t) = 2 - e^{-t} - \cos(t) \quad t \geq 0}$$ step response

b) use convolution shift property

$$x(t-c) * v(t) = w(t-c)$$

where $x(t) * v(t) = w(t)$

and distributivity property to get:

$$y(t) = h(t) * [u(t) - u(t-2)]$$

$$= h(t) * u(t) - h(t) * u(t-2)$$

$$= g(t) - g(t-2)$$

$$\boxed{\begin{aligned} y(t) &= \left(2 - e^{-t} - \cos(t)\right)u(t) \\ &\quad - \left(2 - e^{-(t-2)} - \cos(t-2)\right)u(t-2) \end{aligned}}$$