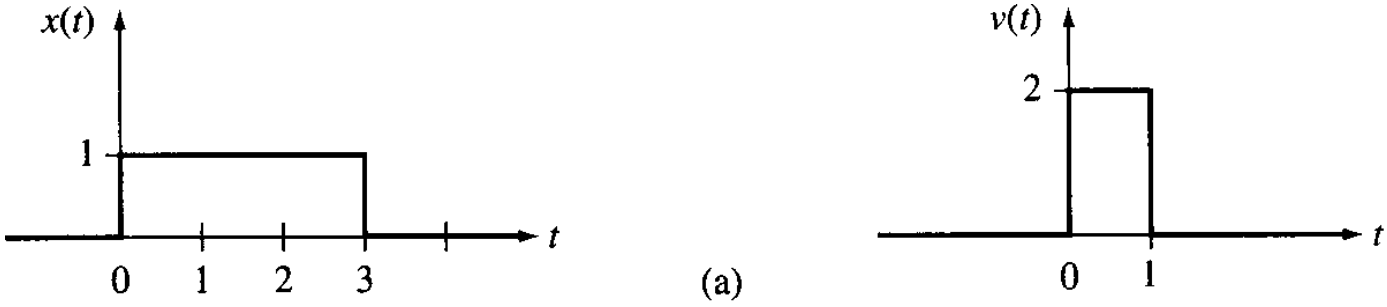
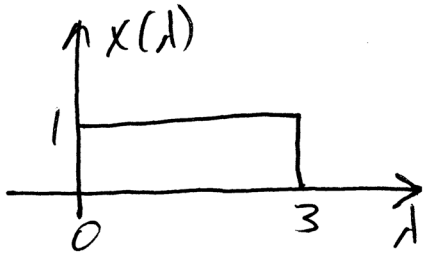
2.29 For the continuous-time signals $x(t)$ and $v(t)$ shown in Figure P2.29, compute the convolution $x(t) * v(t)$ for all $t \geq 0$, and plot your resulting signal.

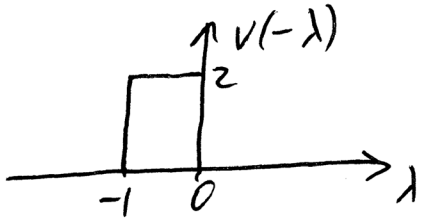


$$(2.72)\quad x(t) * v(t) = \int_{-\infty}^{\infty} x(\lambda)\, v(t-\lambda)\, d\lambda$$

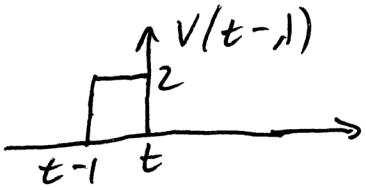
change $t \to -\lambda$

change $t \to \lambda$





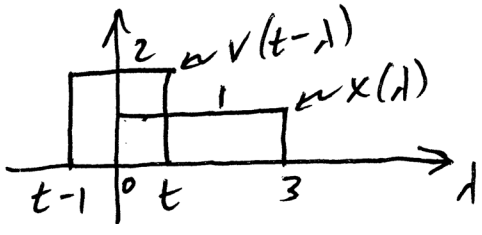
time-shift



1st interval, $t \leq 0$ $x(\lambda) * v(t-\lambda)$ do NOT overlap

$\Rightarrow$ $x(t) * v(t) = 0, \quad t \leq 0$

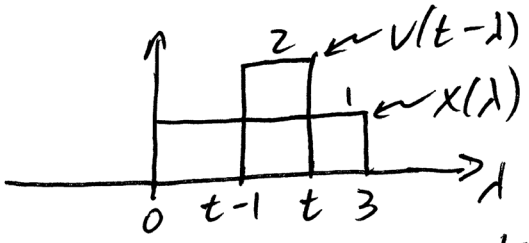
2nd interval $0 \leq t \leq 1$s leading edge of $v(t-\lambda)$ overlaps, trailing edge does NOT overlap



$$x(t) * v(t) = \int_0^t 2(1)\, d\lambda = 2t\Big|_0^t = 2t - 0$$

$$x(t) * v(t) = 2t, \quad 0 \le t \le 1s$$

3rd interval $1 \le t \le 3s$ Full overlap

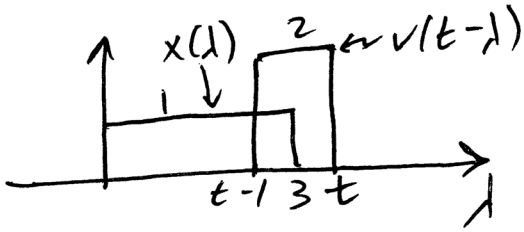


$$x(t) * v(t) = \int_{t-1}^{t} 2(1)\, d\lambda = 2t\Big|_{t-1}^{t}$$

$$= 2t - 2(t-1)$$

$$x(t) * v(t) = 2, \quad 1 \le t \le 3s$$

4th interval, $3 \le t \le 4s$ Trailing edge of $v(t-\lambda)$ overlaps



$$x(t) * v(t) = \int_{t-1}^{3} 2(1)\, d\lambda = 2t\Big|_{t-1}^{3}$$

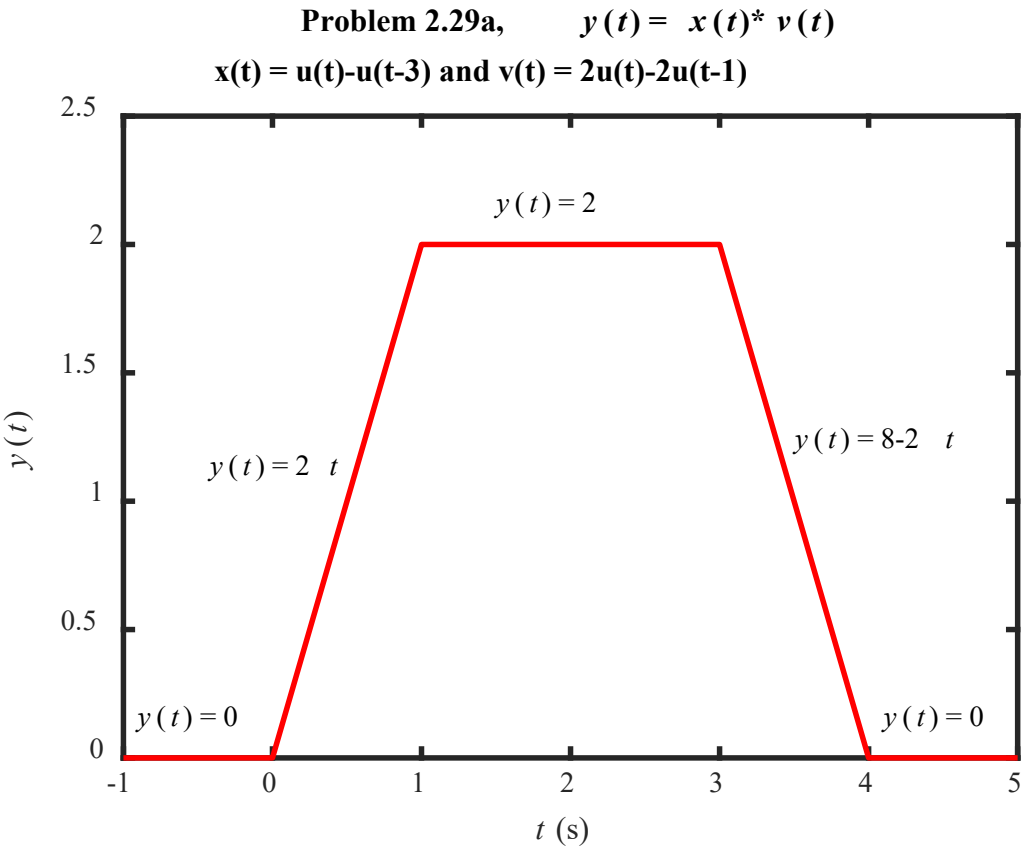
$$= 2(3) - 2(t-1)$$

$$x(t) * v(t) = 8 - 2t, \quad 3 \le t \le 4s$$

5th interval, $t > 4s$ No overlap

$$x(t) * v(t) = 0, \quad t > 4s$$

```
                    % Chapter 2 problem 2.29a (p2_29a.m)
% Plot the convolution of
% x(t) = u(t)-u(t-3) with v(t) = 2u(t)-2u(t-1).
% The analytic result is
%     x(t)*v(t) = 2t for 0 < t < 1 s
%     x(t)*v(t) = 2 for 1 < t < 3 s
%     x(t)*v(t) = 8 - 2t for 3 < t < 4 s
%     x(t)*v(t) = 0 for all other t
clear; clc; close all;
% Calculate the analytic results
t0 = -1:0.1:0; t1 = 0:0.1:1; t2 = 1:0.1:3; t3 = 3:0.1:4; t4 = 4:0.1:5;
y0 = 0*t0; y1 = 2*t1; y2 = 2+0*t2; y3 = 8-2*t3; y4 = 0*t4;
t = [t0 t1 t2 t3 t4]; y = [y0 y1 y2 y3 y4]; % splice together
%
plot(t,y,'r-'), axis([-1 5 0 2.5]),
ylabel('{\ity}({\itt})','fontsize',16,'fontname','times')
xlabel('{\itt} (s)','fontsize',16,'fontname','times')
title({'Problem 2.29a, {\ity}({\itt}) = {\itx}({\itt})*{\itv}({\itt})';...
    'x(t) = u(t)-u(t-3) and v(t) = 2u(t)-2u(t-1)'},...
    'fontsize',18,'fontname','times')
text(-0.9,0.12,'{\ity}({\itt}) = 0','fontsize',14,'fontname','times')
text(-0.42,1.1,'{\ity}({\itt}) = 2{\itt}','fontsize',14,'fontname','times')
text(1.5,2.12,'{\ity}({\itt}) = 2','fontsize',14,'fontname','times')
text(3.5,1.2,'{\ity}({\itt}) = 8-2{\itt}','fontsize',14,'fontname','times')
text(4.1,0.12,'{\ity}({\itt}) = 0','fontsize',14,'fontname','times')
set(findobj('type','line'),'linewidth',2,'markersize',12)
set(findobj('type','axes'),'linewidth',2,'fontsize',14,'fontname','times')
```



2.29 For the continuous-time signals $x(t)$ and $v(t)$ shown in Figure P2.29, compute the convolution $x(t) * v(t)$ for all $t \geq 0$, and plot your resulting signal.

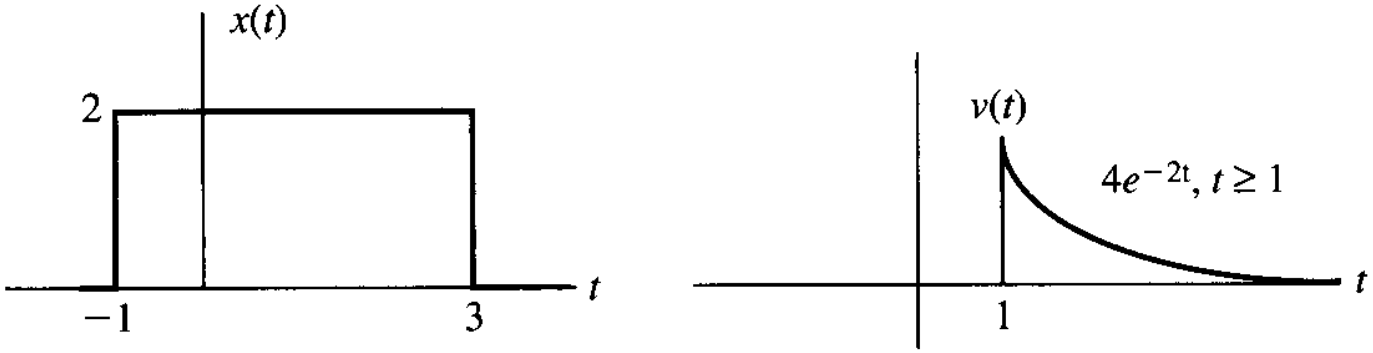
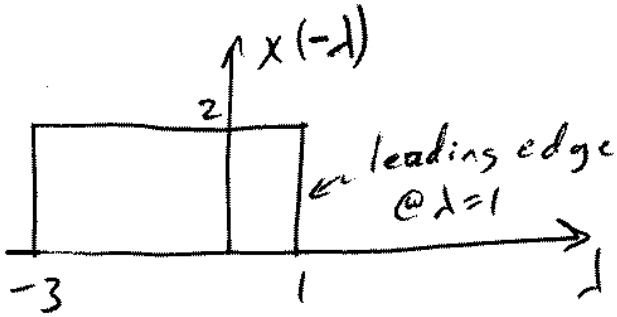
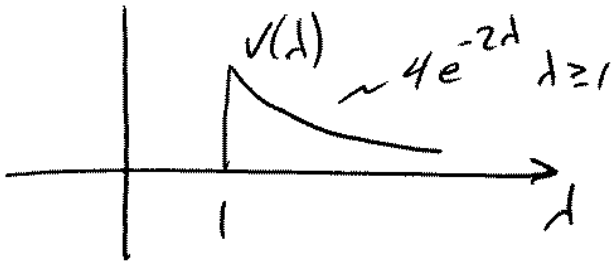


FIGURE P2.29 (e)

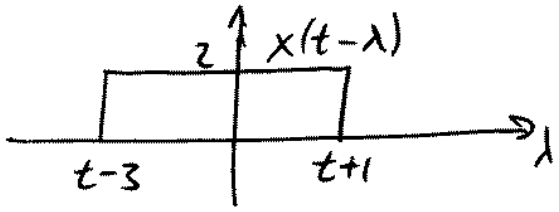
Using commutativity, $x(t) * v(t) = v(t) * x(t) = \int_{-\infty}^{\infty} v(\lambda)\, x(t-\lambda)\, d\lambda$



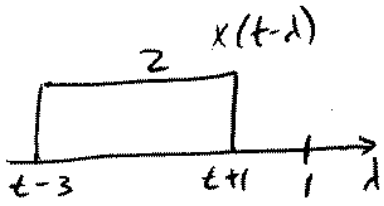


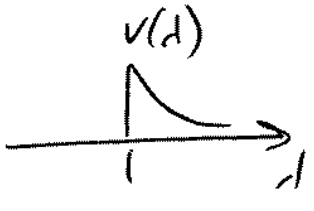
⇓ Time-shift by t

$-\lambda \rightarrow t-\lambda$



1st Interval ($t < 0$)



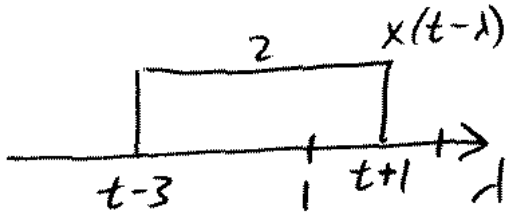


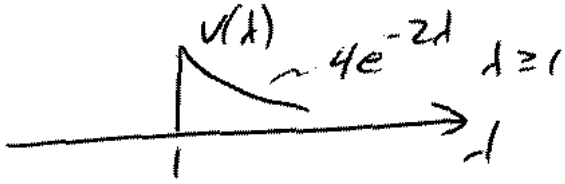
→ $x(t-\lambda)$ and $v(\lambda)$ do Not overlap

$$x(t) * v(t) = 0$$

2nd Interval ($0 < t \leq 4$)

→ leading edge of $x(t-\lambda)$ overlaps $v(\lambda)$, but trailing edge does not overlap



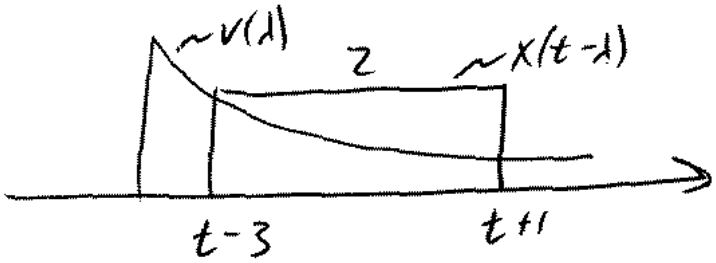


$$x(t) * v(t) = \int_1^{t+1} (2)\,4e^{-2\lambda}\,d\lambda = \frac{8e^{-2\lambda}}{-2}\Bigg|_1^{t+1}$$

$$= -4e^{-2(t+1)} + 4e^{-2(1)} = \underline{4e^{-2}(1-e^{-2t})}$$

3rd Interval ($t \geq 4$)

→ $x(t-\lambda)$ and $v(\lambda)$ fully overlap



$$x(t) * v(t) = \int_{t-3}^{t+1} (2)\,4e^{-2\lambda}\,d\lambda = \frac{8e^{-2\lambda}}{-2}\Bigg|_{t-3}^{t+1}$$

$$= -4e^{-2(t+1)} + 4e^{-2(t-3)}$$

$$= -4e^{-2t}e^{-2} + 4e^{-2t}e^{6}$$

$$= \underline{4e^{-2t}(e^{6} - e^{-2}) \quad t \geq 4}$$

```
% Chapter 2 problem 2.29e (p2_29e.m)
% Plot the convolution of
% x(t) = 2[u(t+1)-u(t-3)] with v(t) = 4e^(-2t)u(t-1).
% The analytic result is
%    x(t) * v(t) = 4e^(-2)[1-e^(-2t)] for 0 < t < 4 s
%    x(t) * v(t) = 4e^(-2t)[e^6-e^-2] for t > 4 s
%    x(t) * v(t) = 0 for all other t
clear;clc;close all;
% Calculate the analytic results
t0 = -3.5:0.05:0; t1 = 0:0.05:4; t2 = 4:0.05:9;
y0 = 0*t0;
y1 = 4*exp(-2)*(1-exp(-2*t1));
y2 = 4*exp(-2*t2)*(exp(6)-exp(-2));
t = [t0 t1 t2]; y = [y0 y1 y2];
%
plot(t,y,'r-'), axis([-3.5 9 0 0.6]),
ylabel('{\ity}({\itt})','fontsize',16,'fontname','times')
xlabel('{\itt} (s)','fontsize',16,'fontname','times')
title({'Problem 2.29e y(t) = x(t)*v(t)';...
    'x(t) = 2[u(t+1)-u(t-3)] and v(t) = 4e^{-2t}u(t-1)'},...
    'fontsize',18,'fontname','times')
text(-3,0.45,'y(t) = 4e^{-2}[1-e^{-2t}]','fontsize',14,'fontname','times')
text(4.5,0.45,'y(t) = 4e^{-2t}[e^{6}-e^{-2}]','fontsize',14,'fontname','times')
set(findobj('type','line'),'linewidth',2,'markersize',12)
set(findobj('type','axes'),'linewidth',2,'fontsize',14,'fontname','times')
```