2.6 **Compute the unit-pulse response $h[n]$ for all integers $n \geq 0$ for each of the following discrete-time systems:**

(d) $y[n+1] - 1/2y[n] = x[n+1] + 1/2x[n]$

- Find an analytic solution.

Exploit time-invariance to re-index the I/O diff. equation, i.e. let $n \to n-1$

$$y[n] - \tfrac{1}{2}y[n-1] = x[n] + \tfrac{1}{2}x[n-1]$$

$$y[n] = \tfrac{1}{2}y[n-1] + x[n] + \tfrac{1}{2}x[n-1]$$

For $x[n] = \delta[n]$, $y[n] = h[n]$ w/ $h[n<0] = 0$

$$h[n] = \tfrac{1}{2}h[n-1] + \delta[n] + \tfrac{1}{2}\delta[n-1]$$

$n=0$ $\quad h[0] = \tfrac{1}{2}\underbrace{h[-1]}_{\to 0} + \underbrace{\delta[0]}_{\to 1} + \tfrac{1}{2}\underbrace{\delta[-1]}_{\to 0} = 1$

$n=1$ $\quad h[1] = \tfrac{1}{2}\underbrace{h[0]}_{\to 1} + \underbrace{\delta[1]}_{\to 0} + \tfrac{1}{2}\underbrace{\delta[0]}_{\to 1} = \tfrac{1}{2}(1) + \tfrac{1}{2}(1) = 1$

$n=2$ $\quad h[2] = \tfrac{1}{2}h[1] + \underbrace{\delta[2]}_{\to 0} + \tfrac{1}{2}\underbrace{\delta[1]}_{\to 0} = \tfrac{1}{2}\left[\tfrac{1}{2}(1) + \tfrac{1}{2}(1)\right] = \tfrac{1}{2}$

$n=3$ $\quad h[3] = \tfrac{1}{2}h[2] + \underbrace{\delta[3]}_{\to 0} + \tfrac{1}{2}\underbrace{\delta[2]}_{\to 0} = \tfrac{1}{2}\left(\tfrac{1}{2}\right) = \tfrac{1}{4}$

See the pattern

$$\boxed{h[n] = \begin{cases} 1 & n = 0 \\ \left(\tfrac{1}{2}\right)^{n-1} = 2\left(\tfrac{1}{2}\right)^n & n \geq 1 \\ 0 & n < 0 \text{ (By def'n)} \end{cases}}$$