

**1.29** Determine whether the following discrete-time systems are causal or noncausal, have memory or are memoryless, are linear or nonlinear, are time invariant or time varying. Justify your answers. In the following parts,  $x[n]$  is an arbitrary input and  $y[n]$  is the response to  $x[n]$ .

(a)  $y[n] = x[n] + 2x[n - 2]$

Causal —  $y[n]$  depends on current + past values of the input

memory —  $y[n]$  depends on  $x[n-2]$ , a past value

linear for  $x[n] = x_1[n]$ ,  $y_1[n] = x_1[n] + 2x_1[n-2]$   
for  $x[n] = x_2[n]$ ,  $y_2[n] = x_2[n] + 2x_2[n-2]$

output for  $x[n] = x_1[n] + x_2[n]$  is

$$\tilde{y}[n] = (x_1[n] + x_2[n]) + 2(x_1[n-2] + x_2[n-2]) = y_1[n] + y_2[n]$$

↳ additive

output for input  $ax[n]$  is

$$\tilde{y}[n] = (ax[n]) + 2(ax[n-2]) = ay[n]$$

↳ homogeneous

time-invariant — if input is  $x[n-n_1]$ , the

$$\text{output } \tilde{y}[n] = x[n-n_1] + 2x[n-n_1-2] = y[n-n_1]$$

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(d)  $y[n] = u[n]x[n]$

Causal -  $y[n]$  depends only on the current value of  $x[n]$ , no future values

Memoryless -  $y[n]$  depends only on current value of  $x[n]$

For  $x_1[n] = x_1[n]$ ,  $y_1[n] = u[n]x_1[n]$

For  $x_2[n] = x_2[n]$ ,  $y_2[n] = u[n]x_2[n]$

For  $x[n] = x_1[n] + x_2[n]$ ,  $\tilde{y}[n] = u[n](x_1[n] + x_2[n])$

Therefore

$$y_1[n] + y_2[n] = u[n]x_1[n] + u[n]x_2[n] = \tilde{y}[n]$$

$\Rightarrow$  additive

For  $x[n] = a x[n]$ ,  $\tilde{y}[n] = u[n](a x[n]) = a u[n] x[n]$

Therefore,  $a y[n] = a u[n] x[n] = \tilde{y}[n] \Rightarrow$  homogeneous

Linear since  $y[n]$  is additive + homogeneous

Time-varying For input  $x[n-1]$ ,  $\tilde{y}[n] = u[n]x[n-1]$

However,  $y[n-1] = u[n-1]x[n-1] \neq \tilde{y}[n]$