1.27   Prove that the following system is linear:

$$y[n] = \sum_{i=0}^{n} a_i x[n - i]$$

where the coefficients a_i are constants.

First, check to see if $y[n]$ is additive:

For input $x_1[n]$, $y_1[n] = \sum_{i=0}^{n} a_i\, x_1[n-i]$.

For input $x_2[n]$, $y_2[n] = \sum_{i=0}^{n} a_i\, x_2[n-i]$.

For input $x_1[n] + x_2[n]$,

$$\tilde{y}[n] = \sum_{i=0}^{n} a_i \left(x_1[n-i] + x_2[n-i] \right) = \sum_{i=0}^{n} a_i x_1[n-i] + a_i x_2[n-i]$$

$$= \sum_{i=0}^{n} a_i\, x_1[n-i] + \sum_{i=0}^{n} a_i\, x_2[n-i] = y_1[n] + y_2[n]$$

$\Rightarrow$ **$y[n]$ is additive**.

Next, check to see if $y[n]$ is homogeneous:

For input $a\, x[n]$, $\tilde{y}[n] = \sum_{i=0}^{n} a_i \left(a\, x[n-i] \right) = a \sum_{i=0}^{n} a_i x[n-i] = a\, y[n]$.

$\Rightarrow$ **$y[n]$ is homogeneous**.

Since $y[n]$ is additive and homogeneous, the **system is linear**.