z-Transform

$$X(z)=\sum_{n=0}^{\infty} x[n]\,z^{-n} \text{ (one-sided) } \;\&\; x[n]=\frac{1}{j\,2\pi}\oint_c X(z)\,z^{n-1}\,dz, \quad (c \text{ in region of convergence})$$

z-Transform Properties

Linearity	$a\,x[n]+b\,v[n] \leftrightarrow a\,X(z)+b\,V(z)$
Right shift of $x[n]\,u[n]$	$x[n-q]\,u[n-q] \leftrightarrow X(z)\,z^{-q}$
Right shift of $x[n]$	$x[n-1] \leftrightarrow X(z)\,z^{-1}+x[-1]$
	$x[n-2] \leftrightarrow X(z)\,z^{-2}+x[-2]+x[-1]\,z^{-1}$
	$\vdots$
	$x[n-q] \leftrightarrow X(z)\,z^{-q}+x[-q]+x[-q+1]\,z^{-1}+\ldots\,x[-1]\,z^{-q+1}$
Left shift of $x[n]$	$x[n+1] \leftrightarrow X(z)\,z-x[0]\,z$
	$x[n+2] \leftrightarrow X(z)\,z^{2}-x[0]\,z^{2}-x[1]\,z$
	$\vdots$
	$x[n+q] \leftrightarrow X(z)\,z^{q}-x[0]\,z^{q}-x[1]\,z^{q-1}-\ldots\,x[q-1]\,z$
Multiplication by n	$n\,x[n] \leftrightarrow -z\dfrac{d\,X(z)}{dz}$
Multiplication by n^2	$n^2\,x[n] \leftrightarrow z\dfrac{d\,X(z)}{dz}+z^2\dfrac{d^2\,X(z)}{dz^2}$
Multiplication by a^n	$x[n]\,a^n \leftrightarrow X\left(\dfrac{z}{a}\right)$
Multiplication by $\cos(\Omega n)$	$x[n]\cos(\Omega n) \leftrightarrow \tfrac{1}{2}\left[X(e^{j\Omega}z)+X(e^{-j\Omega}z)\right]$
Multiplication by $\sin(\Omega n)$	$x[n]\sin(\Omega n) \leftrightarrow \tfrac{j}{2}\left[X(e^{j\Omega}z)-X(e^{-j\Omega}z)\right]$
Summation	$\sum_{i=0}^{n} x[i] \leftrightarrow \dfrac{z}{z-1}X(z)$
Convolution in time-domain	$x[n]*v[n] \leftrightarrow X(z)V(z)$
Initial-value theorem	$x[0]=\lim_{z\to\infty} X(z)$
	$x[1]=\lim_{z\to\infty}\left[z\,X(z)-z\,x[0]\right]$
	$\vdots$
	$x[q]=\lim_{z\to\infty}\left[z^{q}X(z)-z^{q}x[0]-z^{q-1}x[1]-\ldots\,z\,x[q-1]\right]$
Final-value theorem	$\lim_{n\to\infty} x[n]=\left[(z-1)X(z)\right]\Big\vert_{z=1}$, if $X(z)$ is rational and the poles of $(z\text{-}1)X(z)$ have magnitudes less than 1.

z-Transform

$$X(z)=\sum_{n=0}^{\infty} x[n]\,z^{-n} \text{ (one-sided) } \& \; x[n]=\frac{1}{j\,2\pi}\oint_c X(z)\,z^{n-1}\,dz, \quad (c \text{ in region of convergence})$$

z-Transform pairs $\left(x[n] \longleftrightarrow X(z) \right)$

$\delta[n] \leftrightarrow 1$

$\delta[n-q] \leftrightarrow z^{-q}, \quad q=1,2,...$

$u[n] \leftrightarrow \dfrac{z}{z-1}$

$u[n-q] \leftrightarrow z^{-q}\dfrac{z}{(z-1)} = \dfrac{1}{z^{q-1}(z-1)}, \quad q=1,2,\ldots$

$u[n]+u[n-q] \leftrightarrow \dfrac{z^q+1}{z^{q-1}(z-1)}, \quad q=1,2,\ldots$

$u[n]-u[n-q] \leftrightarrow \dfrac{z^q-1}{z^{q-1}(z-1)}, \quad q=1,2,\ldots$

$n\,u[n] \leftrightarrow \dfrac{z}{(z-1)^2}$

$n^2\,u[n] \leftrightarrow \dfrac{z(z+1)}{(z-1)^3}$

$(n+1)\,u[n] \leftrightarrow \dfrac{z^2}{(z-1)^2}$

$n^2 a^n u[n] \leftrightarrow \dfrac{a\,z(z+a)}{(z-a)^3}, \quad a$ can be real or complex

$n(n+1)a^n u[n] \leftrightarrow \dfrac{2a\,z^2}{(z-a)^3}, \quad a$ can be real or complex

$a^n u[n] \leftrightarrow \dfrac{z}{z-a}, \quad a$ can be real or complex

$n\,a^n u[n] \leftrightarrow \dfrac{a\,z}{(z-a)^2}, \quad a$ can be real or complex or $n\,a^{n-1}u[n] \leftrightarrow \dfrac{z}{(z-a)^2}, \quad a$ can be real or complex

$\dfrac{1}{2}n(n-1)\,a^{n-2}u[n-1] \leftrightarrow \dfrac{z}{(z-a)^3}, \quad a$ can be real or complex

$\dfrac{1}{(i-1)!}n(n-1)\cdots(n-i+2)\,a^{n-i-1}u[n-i+2] \leftrightarrow \dfrac{z}{(z-a)^i}, \quad i=4,5,6,\ldots \;\&\; a$ can be real or complex

$\cos(\Omega n)u[n] \leftrightarrow \dfrac{z^2-(\cos\Omega)z}{z^2-(2\cos\Omega)z+1}$

$\sin(\Omega n)u[n] \leftrightarrow \dfrac{(\sin\Omega)z}{z^2-(2\cos\Omega)z+1}$

$a^n\cos(\Omega n)u[n] \leftrightarrow \dfrac{z^2-(a\cos\Omega)z}{z^2-(2a\cos\Omega)z+a^2}$

$a^n\sin(\Omega n)u[n] \leftrightarrow \dfrac{(a\sin\Omega)z}{z^2-(2a\cos\Omega)z+a^2}$