

Continuous-Time Fourier Transform

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt, \quad -\infty < \omega < \infty \quad \text{and} \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega, \quad -\infty < t < \infty$$

Continuous-Time Fourier Transform Properties

Linearity	$a x(t) + b v(t) \leftrightarrow a X(\omega) + b V(\omega)$
Right/left time shift	$x(t - c) \leftrightarrow X(\omega) e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{ a } X\left(\frac{\omega}{a}\right), \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by t^n	$t^n x(t) \leftrightarrow (j)^n \frac{d^n X(\omega)}{d\omega^n}, \quad n = 1, 2, \dots$
Multiplication by $e^{j\omega_0 t}$	$x(t) e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0), \quad \omega_0 \text{ real}$
Multiplication by $\sin(\omega_0 t)$	$x(t) \sin(\omega_0 t) \leftrightarrow \frac{j}{2} [X(\omega + \omega_0) - X(\omega - \omega_0)]$
Multiplication by $\cos(\omega_0 t)$	$x(t) \cos(\omega_0 t) \leftrightarrow \frac{1}{2} [X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation wrt time	$\frac{d^n x(t)}{dt^n} \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \dots$
Integration	$\int_{-\infty}^t x(\lambda) d\lambda \leftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0) \delta(\omega)$
Convolution in time-domain	$x(t) * v(t) \leftrightarrow X(\omega) V(\omega)$
Multiplication in time-domain	$x(t) v(t) \leftrightarrow \frac{1}{2\pi} X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t) v(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{X(\omega)} V(\omega) d\omega$
Special case of Parseval's theorem	$\int_{-\infty}^{\infty} x^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) ^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

Continuous-Time Fourier Transform pairs [i.e., $x(t) \leftrightarrow X(\omega)$]

1, $-\infty < t < \infty \leftrightarrow 2\pi\delta(\omega)$	$-0.5 + u(t) \leftrightarrow \frac{1}{j\omega}$
$u(t) \leftrightarrow \pi \delta(\omega) + \frac{1}{j\omega}$	$\delta(t) \leftrightarrow 1$
$\delta(t - c) \leftrightarrow e^{-j\omega c}, \quad c \text{ real}$	$e^{-bt} u(t) \leftrightarrow \frac{1}{j\omega + b}, \quad b > 0$
$e^{j\omega_0 t} \leftrightarrow 2\pi \delta(\omega - \omega_0), \quad \omega_0 \text{ any real number}$	
$p_\tau(t) \leftrightarrow \tau \operatorname{sinc}\left(\frac{\tau\omega}{2\pi}\right)$	$\tau \operatorname{sinc}\left(\frac{\tau t}{2\pi}\right) \leftrightarrow 2\pi p_\tau(\omega)$
$\Lambda_\tau(t) = \left(1 - \frac{2 t }{\tau}\right) p_\tau(t) \leftrightarrow \frac{\tau}{2} \operatorname{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$	$\frac{\tau}{2} \operatorname{sinc}^2\left(\frac{\tau t}{4\pi}\right) \leftrightarrow 2\pi \left(1 - \frac{2 \omega }{\tau}\right) p_\tau(\omega) = 2\pi \Lambda_\tau(\omega)$
$\cos(\omega_0 t) \leftrightarrow \pi [\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$	$\cos(\omega_0 t + \theta) \leftrightarrow \pi [e^{-j\theta} \delta(\omega + \omega_0) + e^{j\theta} \delta(\omega - \omega_0)]$
$\sin(\omega_0 t) \leftrightarrow j\pi [\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$	$\sin(\omega_0 t + \theta) \leftrightarrow j\pi [e^{-j\theta} \delta(\omega + \omega_0) - e^{j\theta} \delta(\omega - \omega_0)]$