Discrete-Time Fourier Transform (DTFT)

$$X(\Omega)=\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n},\ -\infty \le \Omega \le \infty \text{ periodic @ } 2\pi \text{ intervals} \ \ \& \ \ x[n]=\frac{1}{2\pi}\int_{\Omega_0}^{\Omega_0+2\pi} X(\Omega)\, e^{jn\Omega}\, d\Omega$$

Discrete-Time Fourier Transform Properties

Linearity	$a\,x[n]+b\,v[n] \leftrightarrow a\,X(\Omega)+b\,V(\Omega)$
Right/left time shift	$x(n-q) \leftrightarrow X(\Omega)e^{-jq\Omega},\quad q$ any integer
Time reversal	$x[-n] \leftrightarrow X(-\Omega)=\overline{X(\Omega)}$
Multiplication by n	$n\,x[n] \leftrightarrow j\dfrac{d\,X(\Omega)}{d\Omega}$
Multiplication by $e^{jn\Omega_0}$	$x[n]\,e^{jn\Omega_0} \leftrightarrow X(\Omega-\Omega_0),\quad \Omega_0$ real
Multiplication by $\sin(n\Omega_0)$	$x[n]\sin(\Omega_0\, n) \leftrightarrow \frac{j}{2}\left[X(\Omega+\Omega_0)-X(\Omega-\Omega_0)\right]$
Multiplication by $\cos(n\Omega_0)$	$x[n]\cos(\Omega_0\, n) \leftrightarrow \frac{1}{2}\left[X(\Omega+\Omega_0)+X(\Omega-\Omega_0)\right]$
Convolution in time-domain	$x[n]*v[n] \leftrightarrow X(\Omega)V(\Omega)$
Multiplication in time-domain	$x[n]v[n] \leftrightarrow \dfrac{1}{2\pi}\int_{t_0}^{t_0+2\pi} X(\Omega-\lambda)V(\lambda)\,d\lambda,\quad$ any 2π interval
Summation	$\sum_{i=0}^{n} x[i] \leftrightarrow \dfrac{1}{1-e^{-j\Omega}}X(\Omega)+\sum_{n=-\infty}^{\infty}\pi\, X(2\pi n)\,\delta(\Omega-2\pi n)$
Parseval's theorem	$\sum_{n=-\infty}^{\infty} x[n]v[n]=\frac{1}{2\pi}\int_{t_0}^{t_0+2\pi}\overline{X(\Omega)}\,V(\Omega)\,d\Omega,\quad$ any 2π interval
Special case of Parseval's theorem	$\sum_{n=-\infty}^{\infty} x^2[n]=\frac{1}{2\pi}\int_{t_0}^{t_0+2\pi}\left\lvert X(\Omega)\right\rvert^2 d\Omega,\quad$ any 2π interval
Relationship to inverse CTFT	If $x[n]\leftrightarrow X(\Omega)$ and $\gamma(t)\leftrightarrow X(\omega)\,p_{2\pi}(\omega)$, then $x[n]=\gamma(t)\big\rvert_{t=n}=\gamma(n)$

Discrete-Time Fourier Transform pairs $\left(\ x[n] \quad\leftrightarrow\quad X(\Omega)\ \right)$

$1\,(-\infty<n<\infty) \ \leftrightarrow\ \sum_{k=-\infty}^{\infty} 2\pi\,\delta(\Omega-2\pi k)$

$\delta[n]\leftrightarrow 1$

$u[n]\leftrightarrow \dfrac{1}{1-e^{-j\Omega}}+\sum_{k=-\infty}^{\infty}\pi\,\delta(\Omega-2\pi k)$

$\operatorname{sgn}[n]\leftrightarrow \dfrac{2}{1-e^{-j\Omega}}$, where $\operatorname{sgn}[n]=\begin{cases}1 & n=0,1,2,\ldots\\ -1 & n=-1,-2,\ldots\end{cases}$

$\delta[n-N]\leftrightarrow e^{-jN\Omega},\quad N=\pm1,\pm2,\ldots$

$e^{j\Omega_0 n}\ \leftrightarrow \sum_{k=-\infty}^{\infty} 2\pi\,\delta(\Omega-\Omega_0-2\pi k)$

$a^n u[n]\leftrightarrow \dfrac{1}{1-a\,e^{-j\Omega}},\quad |a|<1$

$\dfrac{B}{\pi}\operatorname{sinc}\left(Bn/\pi\right)\leftrightarrow \sum_{k=-\infty}^{\infty} p_{2B}(\Omega+2\pi k)$

$p_L[n]=p_{2q+1}[n]\leftrightarrow \dfrac{\sin\left[\left(q+\frac{1}{2}\right)\Omega\right]}{\sin(\Omega/2)}$ (Rect. Pulse)

$\cos(\Omega_0 n)\leftrightarrow \sum_{k=-\infty}^{\infty}\pi\left[\delta(\Omega+\Omega_0-2\pi k)+\delta(\Omega-\Omega_0-2\pi k)\right]$

$\sin(\Omega_0 n)\leftrightarrow \sum_{k=-\infty}^{\infty} j\pi\left[\delta(\Omega+\Omega_0-2\pi k)-\delta(\Omega-\Omega_0-2\pi k)\right]$

$$\cos\left(\Omega_0 n+\theta\right)\leftrightarrow \sum_{k=-\infty}^{\infty}\pi\left[e^{-j\theta}\delta\left(\Omega+\Omega_0-2\pi k\right)+e^{j\theta}\delta\left(\Omega-\Omega_0-2\pi k\right)\right]$$

$$\sin\left(\Omega_0 n+\theta\right)\leftrightarrow \sum_{k=-\infty}^{\infty} j\pi\left[e^{-j\theta}\delta\left(\Omega+\Omega_0-2\pi k\right)-e^{j\theta}\delta\left(\Omega-\Omega_0-2\pi k\right)\right]$$