

Discrete-Time Fourier Transform (DTFT)

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}, \quad \text{periodic with period } 2\pi \quad \text{and} \quad x[n] = \frac{1}{2\pi} \int_{t_0}^{t_0+2\pi} X(\Omega) e^{jn\Omega} d\Omega, \quad \text{any } 2\pi \text{ interval}$$

Discrete-Time Fourier Transform Properties

Linearity	$a x[n] + b v[n] \leftrightarrow a X(\Omega) + b V(\Omega)$
Right/left time shift	$x(n-q) \leftrightarrow X(\Omega) e^{-jq\Omega}, \quad q \text{ any integer}$
Time reversal	$x[-n] \leftrightarrow X(-\Omega) = \overline{X(\Omega)}$
Multiplication by n	$n x[n] \leftrightarrow j \frac{dX(\Omega)}{d\Omega}$
Multiplication by $e^{jn\Omega_0}$	$x[n] e^{jn\Omega_0} \leftrightarrow X(\Omega - \Omega_0), \quad \Omega_0 \text{ real}$
Multiplication by $\sin(n\Omega_0)$	$x[n] \sin(\Omega_0 n) \leftrightarrow \frac{j}{2} [X(\Omega + \Omega_0) - X(\Omega - \Omega_0)]$
Multiplication by $\cos(n\Omega_0)$	$x[n] \cos(\Omega_0 n) \leftrightarrow \frac{1}{2} [X(\Omega + \Omega_0) + X(\Omega - \Omega_0)]$
Convolution in time-domain	$x[n] * v[n] \leftrightarrow X(\Omega) V(\Omega)$
Multiplication in time-domain	$x[n] v[n] \leftrightarrow \frac{1}{2\pi} \int_{t_0}^{t_0+2\pi} X(\Omega - \lambda) V(\lambda) d\lambda, \quad \text{any } 2\pi \text{ interval}$
Summation	$\sum_{i=0}^n x[i] \leftrightarrow \frac{1}{1 - e^{-j\Omega}} X(\Omega) + \sum_{n=-\infty}^{\infty} \pi X(2\pi n) \delta(\Omega - 2\pi n)$
Parseval's theorem	$\sum_{n=-\infty}^{\infty} x[n] v[n] = \frac{1}{2\pi} \int_{t_0}^{t_0+2\pi} \overline{X(\Omega)} V(\Omega) d\Omega, \quad \text{any } 2\pi \text{ interval}$
Special case of Parseval's theorem	$\sum_{n=-\infty}^{\infty} x^2[n] = \frac{1}{2\pi} \int_{t_0}^{t_0+2\pi} X(\Omega) ^2 d\Omega, \quad \text{any } 2\pi \text{ interval}$
Relationship to inverse CTFT	If $x[n] \leftrightarrow X(\Omega)$ and $\gamma(t) \leftrightarrow X(\omega) p_{2\pi}(\omega)$, then $x[n] = \gamma(t) _{t=n} = \gamma(n)$

Discrete-Time Fourier Transform pairs ($x[n] \leftrightarrow X(\Omega)$)

$1, \quad -\infty < n < \infty \leftrightarrow \sum_{k=-\infty}^{\infty} 2\pi \delta(\Omega - 2\pi k)$	$\delta[n] \leftrightarrow 1$
$u[n] \leftrightarrow \frac{1}{1 - e^{-j\Omega}} + \sum_{k=-\infty}^{\infty} \pi \delta(\Omega - 2\pi k)$	$\text{sgn}[n] \leftrightarrow \frac{2}{1 - e^{-j\Omega}}, \quad \text{where } \text{sgn}[n] = \begin{cases} 1 & n = 0, 1, 2, \dots \\ -1 & n = -1, -2, \dots \end{cases}$
$\delta[n - N] \leftrightarrow e^{-jN\Omega}, \quad N = \pm 1, \pm 2, \dots$	$e^{j\Omega_0 n} \leftrightarrow \sum_{k=-\infty}^{\infty} 2\pi \delta(\Omega - \Omega_0 - 2\pi k)$
$a^n u[n] \leftrightarrow \frac{1}{1 - a e^{-j\Omega}}, \quad a < 1$	
$\frac{B}{\pi} \text{sinc}\left(\frac{Bn}{\pi}\right) \leftrightarrow \sum_{k=-\infty}^{\infty} p_{2B}(\Omega + 2\pi k)$	$p_L[n] = p_{2q+1}[n] \leftrightarrow \frac{\sin\left[\left(q + \frac{1}{2}\right)\Omega\right]}{\sin(\Omega/2)} \quad (\text{Rect. Pulse})$
$\cos(\Omega_0 n) \leftrightarrow \sum_{k=-\infty}^{\infty} \pi [\delta(\Omega + \Omega_0 - 2\pi k) + \delta(\Omega - \Omega_0 - 2\pi k)]$	$\sin(\Omega_0 n) \leftrightarrow \sum_{k=-\infty}^{\infty} j\pi [\delta(\Omega + \Omega_0 - 2\pi k) - \delta(\Omega - \Omega_0 - 2\pi k)]$
$\cos(\Omega_0 n + \theta) \leftrightarrow \sum_{k=-\infty}^{\infty} \pi \left[e^{-j\theta} \delta(\Omega + \Omega_0 - 2\pi k) + e^{j\theta} \delta(\Omega - \Omega_0 - 2\pi k) \right]$	
$\sin(\Omega_0 n + \theta) \leftrightarrow \sum_{k=-\infty}^{\infty} j\pi \left[e^{-j\theta} \delta(\Omega + \Omega_0 - 2\pi k) - e^{j\theta} \delta(\Omega - \Omega_0 - 2\pi k) \right]$	