Discrete Fourier Transform (DFT)

$$X_k = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \quad k = 0,1,\ldots N-1 \text{ and } x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, \quad n = 0,1,\ldots N-1$$

Discrete Fourier Transform (DFT) Properties

Linearity	$a\,x[n] + b\,v[n] \leftrightarrow a\,X_k + b\,V_k$
Circular time shift	$x(n-q, \bmod N) \leftrightarrow X_k e^{-j2\pi qn/N}$, q any integer
Time reversal	$x[-n, \bmod N] = \begin{cases} x[0] & n = 0 \\ x[N-n] & 0 < n \le N-1 \end{cases} \leftrightarrow X_{-k,\bmod N}$
Multiplication by complex exponential	$x[n]e^{j2\pi qn/N} \leftrightarrow X_{k-q,\bmod N}$
Circular convolution	$x[n] \otimes_N v[n] = \sum_{i=0}^{N-1} x[i]\, v\left[n-i, \bmod N\right] \leftrightarrow X_k\, V_k$
Multiplication in time-domain	$x[n]\, v[n] \leftrightarrow \left(\frac{1}{N}\right) X_k \otimes_N V_k = \left(\frac{1}{N}\right) \sum_{i=0}^{N-1} X_i\, V_{k-i,\bmod N}$
Parseval's theorem	$\sum_{n=0}^{N-1} x[n] v[n] = \left(\frac{1}{N}\right) \sum_{i=0}^{N-1} X_i \overline{V_i}$
Special case of Parseval's theorem	$\sum_{n=0}^{N-1} x^2[n] = \left(\frac{1}{N}\right) \sum_{i=0}^{N-1} X_i \overline{X_i} = \left(\frac{1}{N}\right) \sum_{i=0}^{N-1} \lvert X_i \rvert^2$

Discrete Fourier Transform (DFT) Notes:

1) Mod or modulo operation: DFT is limited to values $0 \le n \le N$-1 (time-domain) and $0 \le k \le N$-1 (frequency-domain). When a value (e.g., indices n and k) is outside this range, the mod or modulo operation "wraps-around" the value to return it to between 0 and N-1.
2) How is the mod or modulo operation defined? For M and N being integers,
   a) Mod(M, N) = remainder of M/N, or, equivalently,
   b) Add/subtract integer multiples of N until the result is within the desired range, i.e., $0 \le$ Mod(M,N) = (M $\pm\, l$ N) $\le N$-1