EE 313 Signals and Systems (Fall 2024) Quiz #8

Name KEY A

Instructions: Open book and notes. Place answers (no fractions) in indicated spaces & **show all work** for credit.

For all n, compute an analytical expression for the unit pulse response $h[n]$ of a causal, linear, time-invariant system with no initial energy characterized by the transfer function

$$H(z) = \frac{z^2 + 0.6z + 0.09}{z^2 - 0.49}.$$

Put $h[n]$ in the simplest possible form. Compute $h[0]$, $h[1]$, $h[5]$, and $h[n \to \infty]$.

Divide $H(z)$ by z to get $\frac{H(z)}{z} = \frac{z^2 + 0.6z + 0.09}{z(z^2 - 0.49)}$. Find roots of denominator & put in partial fraction form- $\frac{H(z)}{z} = \frac{z^2 + 0.6z + 0.09}{z(z-0.7)(z+0.7)} = \frac{C_0}{z} + \frac{C_1}{z-0.7} + \frac{C_2}{z+0.7}$.

Calculate residues:

$$C_0 = \left[z\frac{H(z)}{z} \right]\Bigg|_{z=0} = H(0) = \frac{0^2 + 0.6(0) + 0.09}{(0-0.7)(0+0.7)} = \frac{0.09}{-0.49} = \frac{-9}{49} = -0.1836735$$

$$C_1 = \left[(z-0.7)\frac{H(z)}{z} \right]\Bigg|_{z=0.7} = \left[\frac{z^2 + 0.6z + 0.09}{z(z+0.7)} \right]\Bigg|_{z=0.7} = \frac{(0.7)^2 + 0.6(0.7) + 0.09}{0.7(0.7+0.7)} = \frac{1}{0.98} = 1.02040816$$

$$C_2 = \left[(z+0.7)\frac{H(z)}{z} \right]\Bigg|_{z=-0.7} = \left[\frac{z^2 + 0.6z + 0.09}{z(z-0.7)} \right]\Bigg|_{z=-0.7} = \frac{(-0.7)^2 + 0.6(-0.7) + 0.09}{-0.7(-0.7-0.7)} = \frac{0.16}{0.98} = 0.1632653$$

$$H(z) = C_0 + \frac{C_1 z}{z-0.7} + \frac{C_2 z}{z+0.7} = -0.1837 + \frac{1.0204 z}{z-0.7} + \frac{0.16333 z}{z+0.7}$$

Using transform pairs $\delta[n] \leftrightarrow 1$, $a^n u[n] \leftrightarrow \frac{z}{z-a}$, and the linearity property, yields-

$$\boxed{h[n] = -0.1836735\,\delta[n] + 1.02040816(0.7)^n u[n] + 0.1632653(-0.7)^n u[n]}$$

$$h[0] = -0.1836735(1) + 1.02040816(1)(1) + 0.1632653(1)(1) = 1$$

$$h[1] = -0.1836735(0) + 1.02040816(0.7)(1) + 0.1632653(-0.7)^1(1) = 0.6$$

$$h[5] = -0.1836735(0) + 1.02040816(0.7)^5(1) + 0.1632653(-0.7)^5(1) = 0.14406$$

$$h[\infty] = -0.1836735(0) + 1.02040816(0.7)^\infty(1) + 0.1632653(-0.7)^\infty(1) \to 0$$

***h*[*n*] =** $-0.1836735\,\delta[n] + 1.02040816(0.7)^n u[n] + 0.1632653(-0.7)^n u[n]$

***h*[0] =** 1 ***h*[1] =** 0.6 ***h*[5] =** 0.14406 ***h*[*n* → ∞] =** 0

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Name KEY B

Instructions: Open book and notes. Place answers (no fractions) in indicated spaces & **show all work** for credit.

For all n, compute an analytical expression for the unit pulse response $h[n]$ of a causal, linear, time-invariant system with no initial energy characterized by the transfer function

$$H(z) = \frac{z^2 + 0.8z + 0.16}{z^2 - 0.64}.$$

Put $h[n]$ in the simplest possible form. Compute $h[0]$, $h[1]$, $h[5]$, and $h[n \to \infty]$.

Divide $H(z)$ by z to get $\frac{H(z)}{z} = \frac{z^2 + 0.8z + 0.16}{z(z^2 - 0.64)}$. Find roots of denominator & put in partial fraction form- $\frac{H(z)}{z} = \frac{z^2 + 0.8z + 0.16}{z(z-0.8)(z+0.8)} = \frac{C_0}{z} + \frac{C_1}{z-0.8} + \frac{C_2}{z+0.8}$.

Calculate residues:

$$C_0 = \left[z \frac{H(z)}{z} \right]\Bigg|_{z=0} = H(0) = \frac{0^2 + 0.8(0) + 0.16}{(0-0.8)(0+0.8)} = \frac{0.16}{-0.64} = \frac{-16}{64} = -0.25$$

$$C_1 = \left[(z-0.8) \frac{H(z)}{z} \right]\Bigg|_{z=0.8} = \left[\frac{z^2 + 0.8z + 0.16}{z(z+0.8)} \right]\Bigg|_{z=0.8} = \frac{(0.8)^2 + 0.8(0.8) + 0.16}{0.8(0.8+0.8)} = \frac{1.44}{1.28} = 1.125$$

$$C_2 = \left[(z+0.8) \frac{H(z)}{z} \right]\Bigg|_{z=-0.8} = \left[\frac{z^2 + 0.8z + 0.16}{z(z-0.8)} \right]\Bigg|_{z=-0.8} = \frac{(-0.8)^2 + 0.8(-0.8) + 0.16}{-0.8(-0.8-0.8)} = \frac{0.16}{1.28} = 0.125$$

$$\underline{H(z) = C_0 + \frac{C_1 z}{z-0.8} + \frac{C_2 z}{z+0.8} = -0.25 + \frac{1.125z}{z-0.8} + \frac{0.125z}{z+0.8}}$$

Using transform pairs $\delta[n] \leftrightarrow 1$, $a^n u[n] \leftrightarrow \frac{z}{z-a}$, and the linearity property, yields-

$$\boxed{h[n] = -0.25\delta[n] + 1.125(0.8)^n u[n] + 0.125(-0.8)^n u[n]}$$

$$h[0] = -0.25(1) + 1.125(1)(1) + 0.125(1)(1) = 1$$

$$h[1] = -0.25(0) + 1.125(0.8)(1) + 0.125(-0.8)(1) = 0.8$$

$$h[5] = -0.25(0) + 1.125(0.8)^5(1) + 0.125(-0.8)^5(1) = 0.32768$$

$$h[\infty] = -0.25(0) + 1.125(0.8)^\infty(1) + 0.125(-0.8)^\infty(1) \to 0$$

$\boldsymbol{h[n]}$ **=** $\underline{-0.25\delta[n] + 1.125(0.8)^n u[n] + 0.125(-0.8)^n u[n]}$

$\boldsymbol{h[0]}$ **=** **1** $\boldsymbol{h[1]}$ **=** **0.8** $\boldsymbol{h[5]}$ **=** **0.32768** $\boldsymbol{h[n \to \infty]}$ **=** **0**