

## EE 313 Signals and Systems (Fall 2024) Quiz #7

Name

Key A

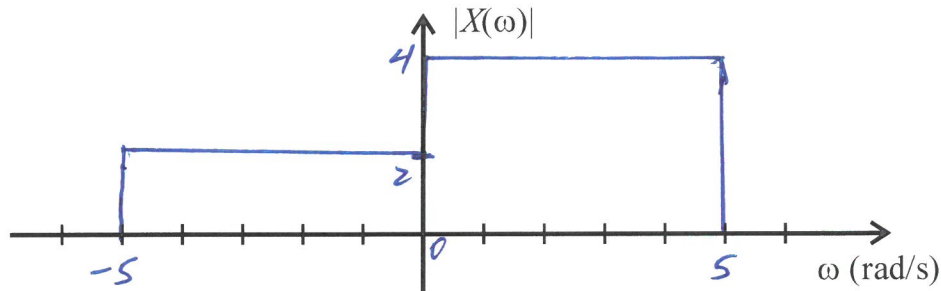
**Instructions:** Open book and notes. Place answers in indicated spaces & show all work for credit.

Cousins Fourier and Nyquist have composed a quiz before Thanksgiving dinner to determine seating. Pass to sit at the adult table. Fail and you will sit with Uncle Laplace's obnoxious brood of loud and querulous brats at a wobbly card table in a cold and dank basement.

A signal  $x(t)$  has the Fourier transform  $X(\omega) = 2p_{10}(\omega) + 2p_5(\omega - 2.5)$ .

- a) Draw a fully labeled sketch of  $|X(\omega)|$  on the provided axes.

$2p_{10}(\omega) \leftarrow 10$  wide, centered @ 0  
 $2p_5(\omega - 2.5) \leftarrow 5$  wide, centered @ 2.5 rad/s



- b) What are the maximum sampling rate  $T_{\max}$  and corresponding minimum sampling frequency  $\omega_{s,\min}$  that will prevent aliasing error?

Per Nyquist sampling theorem,

$$\omega_{s,\min} = 2B = 10 \text{ rad/s}$$

$$T_{\max} = \frac{2\pi}{\omega_{s,\min}} = \frac{2\pi}{10} = \frac{\pi}{5} = 0.62832 \text{ s}$$

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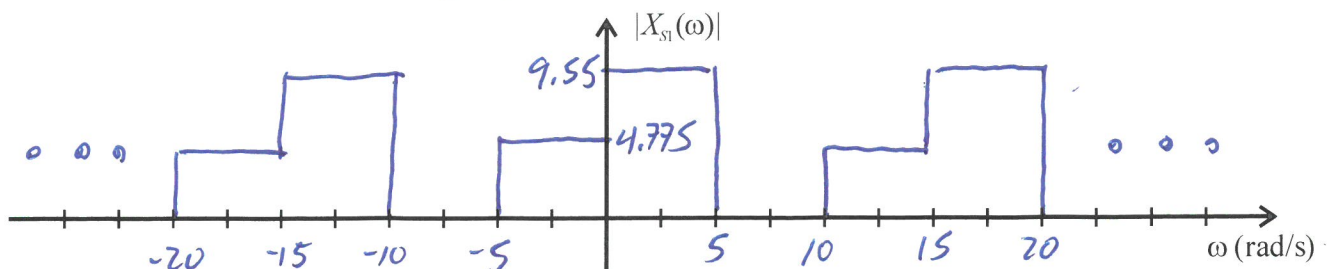
$$\omega_{s,\min} = 10 \text{ rad/s}$$

- c) For a sampling rate  $T_1 = 0.418879 \text{ s}$ , sketch the fully labeled sampled frequency spectrum  $|X_{s1}(\omega)|$  on the provided axes.

$$\omega_{s1} = \frac{2\pi}{T_1} = \frac{2\pi}{0.418879} = 15 \text{ rad/s} > \omega_{s,\min}$$

$$\frac{4}{T_1} = \frac{4}{0.418879} = 9.55 \quad \& \quad \frac{2}{T_1} = 4.775$$

$$(S.51) \quad X_{s1}(\omega) = \frac{1}{T_1} \sum_{k=-\infty}^{\infty} X(\omega - k\omega_{s1})$$



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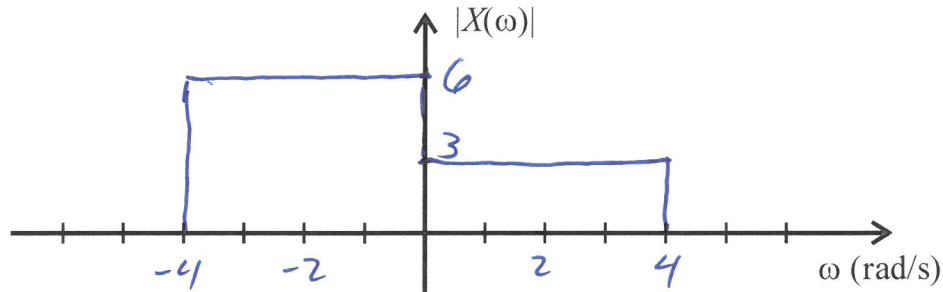
Name Key B**Instructions:** Open book and notes. Place answers in indicated spaces & show all work for credit.

Cousins Fourier and Nyquist have composed a quiz before Thanksgiving dinner to determine seating. Pass to sit at the adult table. Fail and you will sit with Uncle Laplace's dreadful brood of noisy and petulant hellions at a wobbly card table on a cold and drafty porch.

A signal  $x(t)$  has the Fourier transform  $X(\omega) = 3p_8(\omega) + 3p_4(\omega + 2)$ .

- a) Draw a fully labeled sketch of  $|X(\omega)|$  on the provided axes.

$3p_8(\omega) \leftarrow 8 \text{ wide, centered @ } \omega = 0$   
 $3p_4(\omega) \leftarrow 4 \text{ wide, centered @ } \omega = -2$



- b) What are the maximum sampling rate  $T_{\max}$  and corresponding minimum sampling frequency  $\omega_{s,\min}$  that will prevent aliasing error?

Per Nyquist sampling theorem,  
 $\omega_{s,\min} = 2B = 8 \text{ rad/s}$

$$T_{\max} = \frac{2\pi}{\omega_{s,\min}} = \frac{2\pi}{8} = \frac{\pi}{4} = 0.7853985$$

$$T_{\max} = \frac{\pi}{4} = 0.7854 \text{ s} \quad \omega_{s,\min} = 8 \text{ rad/s}$$

- c) For a sampling rate  $T_2 = 0.523599 \text{ s}$ , sketch the fully labeled sampled frequency spectrum  $|X_{s_2}(\omega)|$  on the provided axes.

$$\omega_{s_2} = \frac{2\pi}{T_2} = \frac{2\pi}{0.523599} = 12 \text{ rad/s} > \omega_{s,\min}$$

$$\frac{6}{T_2} = 11.46 \quad \& \quad \frac{3}{T_2} = 5.73$$

$$(5.51) \quad X_{s_2}(\omega) = \frac{1}{T_2} \sum_{k=-\infty}^{\infty} X(\omega - k\omega_{s_2})$$

