Friday, November 15, 2024 A

EE 313 Signals and Systems (Fall 2024) Quiz #6

Name Key A

Instructions: Open book & notes. Place answers in indicated spaces & show all work for credit. Fully label sketches. A continuous signal $x(t) = 10\,\text{sinc}(4t), -\infty < t < \infty$ is input into a filter with the frequency response $H(\omega) = \begin{cases} 0.8 & |\omega| \le 10 \text{ rad/s} \\ 0 & \text{elsewhere} \end{cases}$. Sketch $|H(\omega)|$ and find an equation for $H(\omega)$ in terms of the rectangular pulse function. Find $X(\omega)$ and sketch $|X(\omega)|$. Then, compute the outputs $Y(\omega)$ and $y(t)$ of the filter.

From plot of $|H(\omega)|$, we see $\underline{H(\omega) = 0.8\,p_{20}(\omega)}$

For $X(\omega)$, use linearity and $\tau\,\text{sinc}\left(\frac{\tau t}{2\pi}\right) \leftrightarrow 2\pi\, p_\tau(\omega)$

where $\frac{\tau}{2\pi} = 4 \Rightarrow \tau = 8\pi$

$$10\,\text{sinc}(4t) = \frac{10}{8\pi}\, 8\pi\, \text{sinc}\left(\frac{8\pi t}{2\pi}\right) \leftrightarrow \underline{\frac{10}{8\pi}\, 2\pi\, p_{8\pi}(\omega) = X(\omega)}$$

Use convolution property and sketches $Y(\omega) = X(\omega)H(\omega)$

$$= \underline{0.8(2.5)\, p_{20}(\omega) = 2\,p_{20}(\omega)}$$

For $y(t)$, use linearity and $\tau\,\text{sinc}\left(\frac{\tau t}{2\pi}\right) \leftrightarrow 2\pi\, p_\tau(\omega)$

$$Y(\omega) = 2\,p_{20}(\omega) = \frac{2}{2\pi}\, 2\pi\, p_{20}(\omega)$$

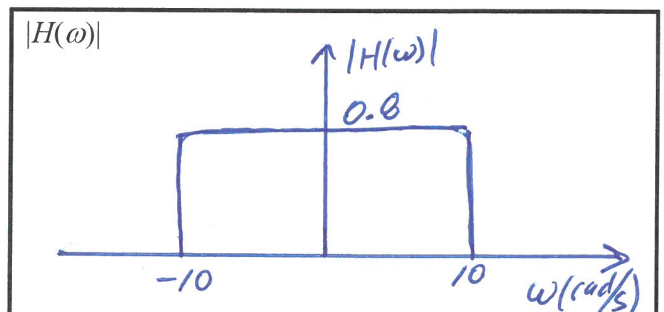
w/ $\tau = 20$

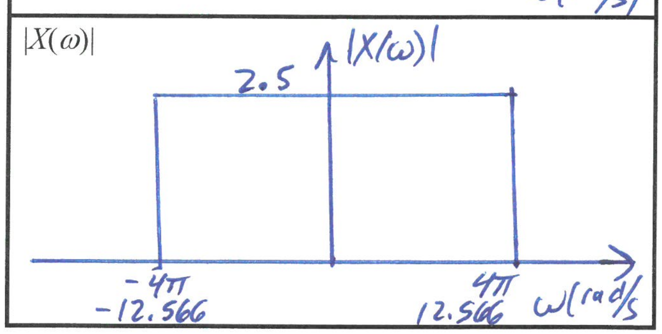
$$y(t) = \frac{2}{2\pi}\, 20\, \text{sinc}\left(\frac{20t}{2\pi}\right)$$

$$= \underline{\frac{20}{\pi}\, \text{sinc}\left(\frac{10t}{\pi}\right)}$$

$$= \underline{6.366\, \text{sinc}(3.183t)}$$

$-\infty < t < \infty$





$H(\omega) =$ $0.8\,p_{20}(\omega)$

$X(\omega) =$ $2.5\,p_{8\pi}(\omega) = 2.5\,p_{25.13}(\omega)$

$Y(\omega) =$ $2\,p_{20}(\omega)$

$y(t) =$ $\frac{20}{\pi}\,\text{sinc}\left(\frac{10t}{\pi}\right)$ $-\infty < t < \infty$

$= \underline{6.366\,\text{sinc}(3.183t)}$ $-\infty < t < \infty$

Friday, November 15, 2024 B

EE 313 Signals and Systems (Fall 2024) Quiz #6

Name Key B

Instructions: Open book & notes. Place answers in indicated spaces & show all work for credit. Fully label sketches.

A continuous signal $x(t) = 12\,\text{sinc}(5t), -\infty < t < \infty$ is input into a filter with the frequency response $H(\omega) = \begin{cases} 0.9 & |\omega| \le 12 \text{ rad/s} \\ 0 & \text{elsewhere} \end{cases}$. Sketch $|H(\omega)|$ and find an equation for $H(\omega)$ in terms of the rectangular pulse function. Find $X(\omega)$ and sketch $|X(\omega)|$. Then, compute the outputs $Y(\omega)$ and $y(t)$ of the filter.

From plot of $|H(\omega)|$, we see $\underline{H(\omega) = 0.9\, p_{24}(\omega)}$

For $X(\omega)$, use linearity + $\tau\,\text{sinc}\left(\frac{\tau t}{2\pi}\right) \Leftrightarrow 2\pi\, p_\tau(\omega)$

where $\frac{\tau}{2\pi} = 5 \Rightarrow \tau = 10\pi$

$$12\,\text{sinc}(5t) = \frac{12}{10\pi}\, 10\pi\,\text{sinc}\left(\frac{10\pi t}{2\pi}\right) \Leftrightarrow \frac{12}{10\pi}\, 2\pi\, p_{10\pi}(\omega) = \underline{2.4\, p_{10\pi}(\omega)} \leftarrow X(\omega)$$

Use convolution property and sketches $Y(\omega) = X(\omega) H(\omega)$

$$= 0.9(2.4)\, p_{24}(\omega) = \underline{2.16\, p_{24}(\omega)}$$

For $y(t)$, use linearity and $\tau\,\text{sinc}\left(\frac{\tau t}{2\pi}\right) \Leftrightarrow 2\pi\, p_\tau(\omega)$

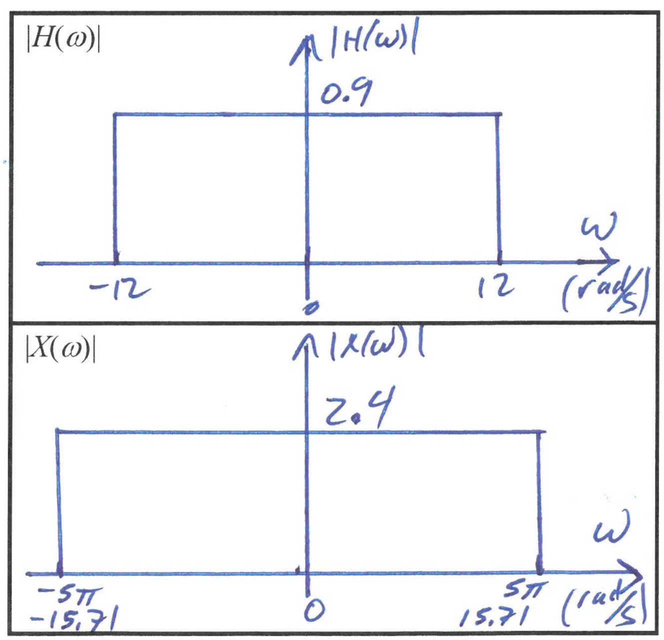
$$Y(\omega) = 2.16\, p_{24}(\omega) = \frac{2.16}{2\pi}\, 2\pi\, p_{24}(\omega)$$

$$y(t) = \frac{2.16}{2\pi}\, 24\,\text{sinc}\left(\frac{24t}{2\pi}\right)$$

$$= \underline{\frac{25.92}{\pi}\,\text{sinc}\left(\frac{12t}{\pi}\right)}$$

$$= \underline{8.2506\,\text{sinc}(3.82t)}$$

$\underline{-\infty < t < \infty}$



$H(\omega) =$ $0.9\, p_{24}(\omega)$

$X(\omega) =$ $2.4\, p_{10\pi}(\omega) = 2.4\, p_{31.416}(\omega)$

$Y(\omega) =$ $2.16\, p_{24}(\omega)$

$y(t) =$ $\frac{25.92}{\pi}\,\text{sinc}\left(\frac{12t}{\pi}\right)$ $-\infty < t < \infty$

$= \underline{8.2506\,\text{sinc}(3.82t) \quad -\infty < t < \infty}$