EE 313 Signals and Systems (Fall 2024) Quiz #5

Name Key A

Instructions: Closed book, notes & homework. Place answers in indicated spaces & show all work for credit.

When $x[n] = 4(0.7)^n u[n]$ and $w[n] = \delta[n-32]$, compute the DTFT of $x[n]$, $w[n]$, and $y[n] = x[n] * w[n]$. Then, find $y[n]$.

To find $X(\Omega)$, use linearity $a x[n] \leftrightarrow a X(\Omega)$ and the transform pair $a^n u[n] \leftrightarrow \frac{1}{1-a e^{-j\Omega}}$

$$4(0.7)^n u[n] \leftrightarrow 4 \frac{1}{1-0.7e^{-j\Omega}} \quad -\infty < \Omega < \infty$$

To find $W(\Omega)$, use transform pair $\delta[n-N] \leftrightarrow e^{-jN\Omega}$

$$\delta[n-32] \leftrightarrow e^{-j32\Omega} \quad -\infty < \Omega < \infty$$

To find $Y(\Omega)$, use "convolution in time-domain" prop.

$$x[n] * w[n] \leftrightarrow X(\Omega) W(\Omega) = Y(\Omega)$$

$$Y(\Omega) = \frac{4}{1-0.7e^{-j\Omega}} e^{-j32\Omega} \quad -\infty < \Omega < \infty$$

To find $y[n]$, use linearity, $a^n u[n] \leftrightarrow \frac{1}{1-a e^{-j\Omega}}$, and the right/left shift property $x[n-q] \leftrightarrow X(\Omega) e^{-jq\Omega}$ w/ $q = 32$

$$y[n] = 4(0.7)^n u[n] \Big|_{n \to n-32}$$

$$= 4(0.7)^{n-32} u[n-32]$$

$X(\Omega) = \frac{4}{1-0.7e^{-j\Omega}} \quad -\infty < \Omega < \infty$

$W(\Omega) = e^{-j32\Omega} \quad -\infty < \Omega < \infty$

$Y(\Omega) = \left(\frac{4}{1-0.7e^{-j\Omega}}\right) e^{-j32\Omega} \quad -\infty < \Omega < \infty$

$y[n] = 4(0.7)^{n-32} u[n-32]$

Discrete-Time Fourier Transform (DTFT)

$$X(\Omega)=\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n},\quad -\infty\le\Omega\le\infty \text{ periodic @ } 2\pi \text{ intervals} \quad \& \quad x[n]=\frac{1}{2\pi}\int_{\Omega_0}^{\Omega_0+2\pi} X(\Omega)\,e^{jn\Omega}\,d\Omega$$

Discrete-Time Fourier Transform Properties

Linearity	$a\,x[n]+b\,v[n]\leftrightarrow a\,X(\Omega)+b\,V(\Omega)$
Right/left time shift	$x(n-q)\leftrightarrow X(\Omega)e^{-jq\Omega}$, q any integer
Time reversal	$x[-n]\leftrightarrow X(-\Omega)=\overline{X(\Omega)}$
Multiplication by n	$n\,x[n]\leftrightarrow j\dfrac{d\,X(\Omega)}{d\Omega}$
Multiplication by $e^{jn\Omega_0}$	$x[n]\,e^{jn\Omega_0}\leftrightarrow X(\Omega-\Omega_0)$, Ω_0 real
Multiplication by $\sin(n\Omega_0)$	$x[n]\sin(\Omega_0 n)\leftrightarrow \frac{j}{2}[X(\Omega+\Omega_0)-X(\Omega-\Omega_0)]$
Multiplication by $\cos(n\Omega_0)$	$x[n]\cos(\Omega_0 n)\leftrightarrow \frac{1}{2}[X(\Omega+\Omega_0)+X(\Omega-\Omega_0)]$
Convolution in time-domain	$x[n]*v[n]\leftrightarrow X(\Omega)V(\Omega)$
Multiplication in time-domain	$x[n]v[n]\leftrightarrow \dfrac{1}{2\pi}\displaystyle\int_{t_0}^{t_0+2\pi} X(\Omega-\lambda)V(\lambda)\,d\lambda$, any 2π interval
Parseval's theorem	$\displaystyle\sum_{n=-\infty}^{\infty} x[n]v[n]=\frac{1}{2\pi}\int_{t_0}^{t_0+2\pi}\overline{X(\Omega)}\,V(\Omega)\,d\Omega$, any 2π interval
Special case of Parseval's theorem	$\displaystyle\sum_{n=-\infty}^{\infty} x^2[n]=\frac{1}{2\pi}\int_{t_0}^{t_0+2\pi}\lvert X(\Omega)\rvert^2\,d\Omega$, any 2π interval
Relationship to inverse CTFT	If $x[n]\leftrightarrow X(\Omega)$ and $\gamma(t)\leftrightarrow X(\omega)p_{2\pi}(\omega)$, then $x[n]=\gamma(t)\vert_{t=n}=\gamma(n)$

Discrete-Time Fourier Transform pairs ($x[n] \leftrightarrow X(\Omega)$)

$1\,(-\infty<n<\infty)\leftrightarrow \displaystyle\sum_{k=-\infty}^{\infty} 2\pi\,\delta(\Omega-2\pi k)$

$\delta[n]\leftrightarrow 1$

$u[n]\leftrightarrow \dfrac{1}{1-e^{-j\Omega}}+\displaystyle\sum_{k=-\infty}^{\infty}\pi\,\delta(\Omega-2\pi k)$

$\mathrm{sgn}[n]\leftrightarrow \dfrac{2}{1-e^{-j\Omega}}$, where $\mathrm{sgn}[n]=\begin{cases}1 & n=0,1,2,\ldots\\ -1 & n=-1,-2,\ldots\end{cases}$

$\delta[n-N]\leftrightarrow e^{-jN\Omega}$, $N=\pm1,\pm2,\ldots$

$e^{j\Omega_0 n}\leftrightarrow \displaystyle\sum_{k=-\infty}^{\infty} 2\pi\,\delta(\Omega-\Omega_0-2\pi k)$

$a^n u[n]\leftrightarrow \dfrac{1}{1-a\,e^{-j\Omega}}$, $\lvert a\rvert<1$

$\dfrac{B}{\pi}\mathrm{sinc}\left(\dfrac{Bn}{\pi}\right)\leftrightarrow \displaystyle\sum_{k=-\infty}^{\infty} p_{2B}(\Omega+2\pi k)$

$p_L[n]=p_{2q+1}[n]\leftrightarrow \dfrac{\sin\left[\left(q+\frac{1}{2}\right)\Omega\right]}{\sin(\Omega/2)}$ (Rect. Pulse)

$\cos(\Omega_0 n)\leftrightarrow \displaystyle\sum_{k=-\infty}^{\infty}\pi[\delta(\Omega+\Omega_0-2\pi k)+\delta(\Omega-\Omega_0-2\pi k)]$

$\sin(\Omega_0 n)\leftrightarrow \displaystyle\sum_{k=-\infty}^{\infty} j\pi[\delta(\Omega+\Omega_0-2\pi k)-\delta(\Omega-\Omega_0-2\pi k)]$

$$\cos(\Omega_0 n+\theta)\leftrightarrow \sum_{k=-\infty}^{\infty}\pi\left[e^{-j\theta}\delta(\Omega+\Omega_0-2\pi k)+e^{j\theta}\delta(\Omega-\Omega_0-2\pi k)\right]$$

$$\sin(\Omega_0 n+\theta)\leftrightarrow \sum_{k=-\infty}^{\infty} j\pi\left[e^{-j\theta}\delta(\Omega+\Omega_0-2\pi k)-e^{j\theta}\delta(\Omega-\Omega_0-2\pi k)\right]$$

☻ Monday, October 28, 2024 ☠ C

EE 313 Signals and Systems (Fall 2024) Quiz #5

Name Key B

Instructions: Closed book, notes & homework. Place answers in indicated spaces & show all work for credit.

When $x[n] = 5(0.8)^n u[n]$ and $v[n] = \delta[n\text{-}24]$, compute the DTFT of $x[n]$, $v[n]$, and $z[n] = x[n] * v[n]$. Then, find $z[n]$.

To find $X(\Omega)$, use linearity $ax[n] \leftrightarrow aX(\Omega)$ and transform pair $a^n u[n] \leftrightarrow \frac{1}{1-ae^{-j\Omega}}$

$$5(0.8)^n u[n] \leftrightarrow 5\frac{1}{1-0.8e^{-j\Omega}} \quad -\infty<\Omega<\infty$$

To find $V(\Omega)$, use transform pair $\delta[n-N] \leftrightarrow e^{-jN\Omega}$

$$\delta[n-24] \leftrightarrow e^{-j24\Omega} \quad -\infty<\Omega<\infty$$

To find $Z(\Omega)$, use 'Convolution in time-domain' prop.

$$z[n] = x[n] * v[n] \leftrightarrow Z(\Omega) = X(\Omega)V(\Omega)$$

$$Z(\Omega) = \frac{5}{1-0.8e^{-j\Omega}} e^{-j24\Omega} \quad -\infty<\Omega<\infty$$

To find $z[n]$, use linearity, $a^n u[n] \leftrightarrow \frac{1}{1-ae^{-j\Omega}}$, and right/left shift property $x[n-q] \leftrightarrow X(\Omega)e^{-jq\Omega}$ w/ $q=24$

$$z[n] = 5(0.8)^n u[n]\Big|_{n\to n-24}$$

$$= 5(0.8)^{n-24} u[n-24]$$

$X(\Omega) = \frac{5}{1-0.8e^{-j\Omega}} \quad -\infty<\Omega<\infty$ $V(\Omega) = e^{-j24\Omega} \quad -\infty<\Omega<\infty$

$Z(\Omega) = \frac{5}{1-0.8e^{-j\Omega}} e^{-j24\Omega} \quad -\infty<\Omega<\infty$ $z[n] = 5(0.8)^{n-24} u[n-24]$

Discrete-Time Fourier Transform (DTFT)

$$X(\Omega)=\sum_{n=-\infty}^{\infty}x[n]e^{-j\Omega n},\quad -\infty\le\Omega\le\infty \text{ periodic @ } 2\pi \text{ intervals} \quad \& \quad x[n]=\frac{1}{2\pi}\int_{\Omega_0}^{\Omega_0+2\pi}X(\Omega)\,e^{jn\Omega}\,d\Omega$$

Discrete-Time Fourier Transform Properties

Linearity	$a\,x[n]+b\,v[n]\leftrightarrow a\,X(\Omega)+b\,V(\Omega)$
Right/left time shift	$x(n-q)\leftrightarrow X(\Omega)e^{-jq\Omega}$, q any integer
Time reversal	$x[-n]\leftrightarrow X(-\Omega)=X(\Omega)$
Multiplication by n	$n\,x[n]\leftrightarrow j\dfrac{d\,X(\Omega)}{d\Omega}$
Multiplication by $e^{jn\Omega_0}$	$x[n]\,e^{jn\Omega_0}\leftrightarrow X(\Omega-\Omega_0)$, Ω_0 real
Multiplication by $\sin(n\Omega_0)$	$x[n]\sin(\Omega_0 n)\leftrightarrow \frac{j}{2}[X(\Omega+\Omega_0)-X(\Omega-\Omega_0)]$
Multiplication by $\cos(n\Omega_0)$	$x[n]\cos(\Omega_0 n)\leftrightarrow \frac{1}{2}[X(\Omega+\Omega_0)+X(\Omega-\Omega_0)]$
Convolution in time-domain	$x[n]*v[n]\leftrightarrow X(\Omega)V(\Omega)$
Multiplication in time-domain	$x[n]v[n]\leftrightarrow \dfrac{1}{2\pi}\displaystyle\int_{t_0}^{t_0+2\pi}X(\Omega-\lambda)V(\lambda)\,d\lambda$, any 2π interval
Parseval's theorem	$\displaystyle\sum_{n=-\infty}^{\infty}x[n]v[n]=\frac{1}{2\pi}\int_{t_0}^{t_0+2\pi}\overline{X(\Omega)}\,V(\Omega)\,d\Omega$, any 2π interval
Special case of Parseval's theorem	$\displaystyle\sum_{n=-\infty}^{\infty}x^2[n]=\frac{1}{2\pi}\int_{t_0}^{t_0+2\pi}\lvert X(\Omega)\rvert^2\,d\Omega$, any 2π interval
Relationship to inverse CTFT	If $x[n]\leftrightarrow X(\Omega)$ and $\gamma(t)\leftrightarrow X(\omega)p_{2\pi}(\omega)$, then $x[n]=\gamma(t)\big\vert_{t=n}=\gamma(n)$

Discrete-Time Fourier Transform pairs ($x[n] \quad\leftrightarrow\quad X(\Omega)$)

$$1\ (-\infty<n<\infty)\ \leftrightarrow\ \sum_{k=-\infty}^{\infty}2\pi\,\delta(\Omega-2\pi k)$$

$$\delta[n]\leftrightarrow 1$$

$$u[n]\leftrightarrow\frac{1}{1-e^{-j\Omega}}+\sum_{k=-\infty}^{\infty}\pi\,\delta(\Omega-2\pi k)$$

$$\operatorname{sgn}[n]\leftrightarrow\frac{2}{1-e^{-j\Omega}},\text{ where }\operatorname{sgn}[n]=\begin{cases}1 & n=0,1,2,\ldots\\ -1 & n=-1,-2,\ldots\end{cases}$$

$$\delta[n-N]\leftrightarrow e^{-jN\Omega},\quad N=\pm1,\pm2,\ldots$$

$$e^{j\Omega_0 n}\leftrightarrow\sum_{k=-\infty}^{\infty}2\pi\,\delta(\Omega-\Omega_0-2\pi k)$$

$$a^n u[n]\leftrightarrow\frac{1}{1-a\,e^{-j\Omega}},\quad |a|<1$$

$$\frac{B}{\pi}\operatorname{sinc}\left(\frac{Bn}{\pi}\right)\leftrightarrow\sum_{k=-\infty}^{\infty}p_{2B}(\Omega+2\pi k)$$

$$p_L[n]=p_{2q+1}[n]\leftrightarrow\frac{\sin\left[\left(q+\frac{1}{2}\right)\Omega\right]}{\sin(\Omega/2)}\quad\text{(Rect. Pulse)}$$

$$\cos(\Omega_0 n)\leftrightarrow\sum_{k=-\infty}^{\infty}\pi[\delta(\Omega+\Omega_0-2\pi k)+\delta(\Omega-\Omega_0-2\pi k)]$$

$$\sin(\Omega_0 n)\leftrightarrow\sum_{k=-\infty}^{\infty}j\pi[\delta(\Omega+\Omega_0-2\pi k)-\delta(\Omega-\Omega_0-2\pi k)]$$

$$\cos(\Omega_0 n+\theta)\leftrightarrow\sum_{k=-\infty}^{\infty}\pi\left[e^{-j\theta}\delta\left(\Omega+\Omega_0-2\pi k\right)+e^{j\theta}\delta\left(\Omega-\Omega_0-2\pi k\right)\right]$$

$$\sin(\Omega_0 n+\theta)\leftrightarrow\sum_{k=-\infty}^{\infty}j\pi\left[e^{-j\theta}\delta\left(\Omega+\Omega_0-2\pi k\right)-e^{j\theta}\delta\left(\Omega-\Omega_0-2\pi k\right)\right]$$