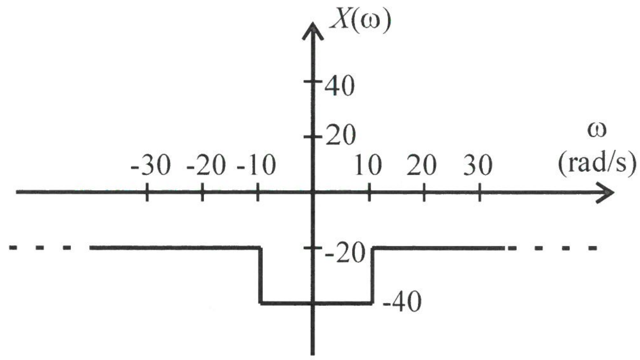
Wednesday, October 23, 2024 A

EE 313 Signals and Systems (Fall 2024) Quiz #4

Name Key A

Instructions: Closed book, notes & homework. Place answers in indicated spaces & show all work for credit.

a) Find a mathematical expression/equation for the signal whose frequency spectra is shown below.



→ constant value of -20 for all ω

→ -20 amplitude rect. pulse of width 20 rad/s

$X(\omega) = -20 - 20\,p_{20}(\omega) \quad -\infty < \omega < \infty$

b) Find the time-domain signal whose frequency spectra is $X(\omega)$ (i.e., find inverse Fourier transform).

Use: $\delta(t) \leftrightarrow 1 \quad -\infty < \omega < \infty$

$\tau \,\text{sinc}\left(\frac{\tau t}{2\pi}\right) \leftrightarrow 2\pi\, p_\tau(\omega)$ w/ $\tau = 20$

& linearity property

to get $-20\,\delta(t) \leftrightarrow -20$

$-20\left(\frac{20}{2\pi}\right)\text{sinc}\left(\frac{20t}{2\pi}\right) \leftrightarrow -20\,p_{20}(\omega)$

$x(t) = -20\left[\delta(t) + \underbrace{\frac{10}{\pi}}_{3.183}\text{sinc}\left(\underbrace{\frac{10}{\pi}}_{3.183} t\right)\right] \quad -\infty < t < \infty$

c) Find the Fourier transform of the signal $v(t) = 12\,\text{sinc}(3t)\cos(10t)$.

Use: $\tau \,\text{sinc}\left(\frac{\tau t}{2\pi}\right) \leftrightarrow 2\pi\, p_\tau(\omega)$ w/ $\frac{\tau}{2\pi} = 3 \Rightarrow \tau = 6\pi$

$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}\left[X(\omega+\omega_0) + X(\omega-\omega_0)\right] \quad \omega_0 = 10$

+ linearity property

To get: $12\,\text{sinc}(3t) = \frac{12}{6\pi} 6\pi\,\text{sinc}\left(\frac{6\pi t}{2\pi}\right) \leftrightarrow \frac{12}{6\pi}2\pi\, p_{6\pi}(\omega) = 4\,p_{6\pi}(\omega)$

& $12\,\text{sinc}(3t)\cos(10t) \leftrightarrow \frac{1}{2}\left[4\,p_{6\pi}(\omega+10) + 4\,p_{6\pi}(\omega-10)\right]$

$V(\omega) = 2\,p_{6\pi}(\omega+10) + 2\,p_{6\pi}(\omega-10) \quad -\infty < \omega < \infty$

$= 2\,p_{18.85}(\omega+10) + 2\,p_{18.85}(\omega-10) \quad -\infty < \omega < \infty$

Continuous-Time Fourier Transform

$$X(\omega)=\int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}\,dt,\quad -\infty<\omega<\infty \quad \text{and} \quad x(t)=\tfrac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\,e^{j\omega t}\,d\omega,\quad -\infty<t<\infty$$

Continuous-Time Fourier Transform Properties

Linearity	$a\,x(t)+b\,v(t)\leftrightarrow a\,X(\omega)+b\,V(\omega)$
Right/left time shift	$x(t-c)\leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at)\leftrightarrow \tfrac{1}{a}X\left(\tfrac{\omega}{a}\right),\quad a>0$
Time reversal	$x(-t)\leftrightarrow X(-\omega)=\overline{X(\omega)}$
Multiplication by t^n	$t^n\,x(t)\leftrightarrow (j)^n\dfrac{d^n X(\omega)}{d\omega^n},\quad n=1,2,\ldots$
Multiplication by $e^{j\omega_0 t}$	$x(t)\,e^{j\omega_0 t}\leftrightarrow X(\omega-\omega_0),\quad \omega_0$ real
Multiplication by $\sin(\omega_0 t)$	$x(t)\sin(\omega_0 t)\leftrightarrow \tfrac{j}{2}\left[X(\omega+\omega_0)-X(\omega-\omega_0)\right]$
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t)\leftrightarrow \tfrac{1}{2}\left[X(\omega+\omega_0)+X(\omega-\omega_0)\right]$
Differentiation wrt time	$\dfrac{d^n x(t)}{dt^n}\leftrightarrow (j\omega)^n X(\omega)\quad n=1,2,\ldots$
Integration	$\int_{-\infty}^{t} x(\lambda)\,d\lambda\leftrightarrow \tfrac{1}{j\omega}X(\omega)+\pi X(0)\delta(\omega)$
Convolution in time-domain	$x(t)*v(t)\leftrightarrow X(\omega)V(\omega)$
Multiplication in time-domain	$x(t)\,v(t)\leftrightarrow \tfrac{1}{2\pi}X(\omega)*V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)\,v(t)\,dt=\tfrac{1}{2\pi}\int_{-\infty}^{\infty}\overline{X(\omega)}\,V(\omega)\,d\omega$
Special case of Parseval's theorem	$\int_{-\infty}^{\infty} x^2(t)\,dt=\tfrac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2\,d\omega$
Duality	$X(t)\leftrightarrow 2\pi x(-\omega)$

Continuous-Time Fourier Transform pairs [i.e., $x(t)\leftrightarrow X(\omega)$]

$1,\quad -\infty<t<\infty\leftrightarrow 2\pi\delta(\omega)$

$-0.5+u(t)\leftrightarrow \dfrac{1}{j\omega}$

$u(t)\leftrightarrow \pi\,\delta(\omega)+\dfrac{1}{j\omega}$

$\delta(t)\leftrightarrow 1$

$\delta(t-c)\leftrightarrow e^{-j\omega c},\quad c$ real

$e^{-bt}u(t)\leftrightarrow \dfrac{1}{j\omega+b},\quad b>0$

$e^{j\omega_0 t}\leftrightarrow 2\pi\,\delta(\omega-\omega_0),\quad \omega_0$ any real number

$p_\tau(t)\leftrightarrow \tau\,\mathrm{sinc}\left(\dfrac{\tau\omega}{2\pi}\right)$

$\tau\,\mathrm{sinc}\left(\dfrac{\tau t}{2\pi}\right)\leftrightarrow 2\pi\,p_\tau(\omega)$

$\Lambda_\tau(t)=\left(1-\dfrac{2|t|}{\tau}\right)p_\tau(t)\leftrightarrow \dfrac{\tau}{2}\mathrm{sinc}^2\left(\dfrac{\tau\omega}{4\pi}\right)$

$\dfrac{\tau}{2}\mathrm{sinc}^2\left(\dfrac{\tau t}{4\pi}\right)\leftrightarrow 2\pi\left(1-\dfrac{2|\omega|}{\tau}\right)p_\tau(\omega)=2\pi\,\Lambda_\tau(\omega)$

$\cos(\omega_0 t)\leftrightarrow \pi\left[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\right]$

$\cos(\omega_0 t+\theta)\leftrightarrow \pi\left[e^{-j\theta}\delta(\omega+\omega_0)+e^{j\theta}\delta(\omega-\omega_0)\right]$

$\sin(\omega_0 t)\leftrightarrow j\pi\left[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)\right]$

$\sin(\omega_0 t+\theta)\leftrightarrow j\pi\left[e^{-j\theta}\delta(\omega+\omega_0)-e^{j\theta}\delta(\omega-\omega_0)\right]$

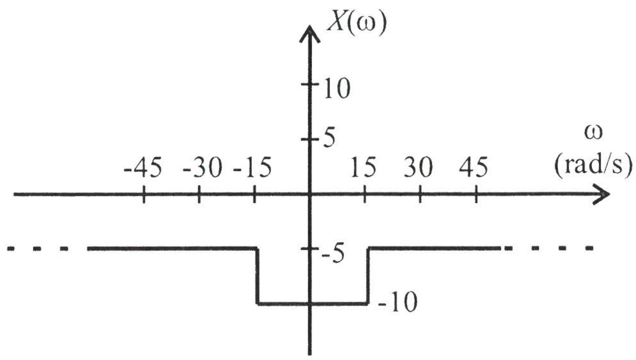
Wednesday, October 23, 2024 C

EE 313 Signals and Systems (Fall 2024) Quiz #4

Name Key B

Instructions: Closed book, notes & homework. Place answers in indicated spaces & show all work for credit.

a) Find a mathematical expression/equation for the signal whose frequency spectra is shown below.



→ constant value of −5 for all ω

plus

→ −5 amplitude rect. pulse of width 30 rad/s

||

$X(\omega) = -5 - 5\,p_{30}(\omega) \quad -\infty < \omega < \infty$

b) Find the time-domain signal whose frequency spectra is $X(\omega)$ (i.e., find inverse Fourier transform).

use: $\delta(t) \leftrightarrow 1 \quad -\infty < \omega < \infty$

$\tau\,\mathrm{sinc}\left(\frac{\tau t}{2\pi}\right) \leftrightarrow 2\pi p_\tau(\omega)$ w/ $\tau = 30$

& linearity property

To get: $-5\,\delta(t) \leftrightarrow -5$

$-5\left(\frac{30}{2\pi}\,\mathrm{sinc}\left(\frac{30t}{2\pi}\right)\right) \leftrightarrow -5\,p_{30}(\omega)$

$x(t) = -5\left[\delta(t) + \underbrace{\frac{15}{\pi}}_{4.775}\mathrm{sinc}\left(\underbrace{\frac{15}{\pi}}_{4.775}t\right)\right] \quad -\infty < t < \infty$

c) Find the Fourier transform of the signal $w(t) = 16\,\mathrm{sinc}(4t)\cos(8t)$.

Use: $\tau\,\mathrm{sinc}\left(\frac{\tau t}{2\pi}\right) \leftrightarrow 2\pi p_\tau(\omega)$ w/ $\frac{\tau}{2\pi} = 4 \Rightarrow \tau = 8\pi$

$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}\left[X(\omega + \omega_0) + X(\omega - \omega_0)\right] \quad \omega_0 = 8$

& linearity property

to get: $16\,\mathrm{sinc}(4t) = \frac{16}{8\pi}\,8\pi\,\mathrm{sinc}\left(\frac{8\pi t}{2\pi}\right) \leftrightarrow \frac{16}{8\pi}\,2\pi\,p_{8\pi}(\omega) = 4\,p_{8\pi}(\omega)$

& $16\,\mathrm{sinc}(4t)\cos(8t) \leftrightarrow \frac{1}{2}\left[4\,p_{8\pi}(\omega+8) + 4\,p_{8\pi}(\omega-8)\right]$

$W(\omega) = 2\,p_{8\pi}(\omega+8) + 2\,p_{8\pi}(\omega-8) \quad -\infty < \omega < \infty$

$= 2\,p_{25.13}(\omega+8) + 2\,p_{25.13}(\omega-8) \quad -\infty < \omega < \infty$

Continuous-Time Fourier Transform

$$X(\omega)=\int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}\,dt,\quad -\infty<\omega<\infty \quad \text{and} \quad x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\,e^{j\omega t}\,d\omega,\quad -\infty<t<\infty$$

Continuous-Time Fourier Transform Properties

Linearity	$a\,x(t)+b\,v(t)\leftrightarrow a\,X(\omega)+b\,V(\omega)$
Right/left time shift	$x(t-c)\leftrightarrow X(\omega)\,e^{-j\omega c}$
Time scaling	$x(at)\leftrightarrow \frac{1}{a}X\left(\frac{\omega}{a}\right),\quad a>0$
Time reversal	$x(-t)\leftrightarrow X(-\omega)=\overline{X(\omega)}$
Multiplication by t^n	$t^n\,x(t)\leftrightarrow (j)^n\,\dfrac{d^n X(\omega)}{d\omega^n},\quad n=1,2,\ldots$
Multiplication by $e^{j\omega_0 t}$	$x(t)\,e^{j\omega_0 t}\leftrightarrow X(\omega-\omega_0),\quad \omega_0$ real
Multiplication by $\sin(\omega_0 t)$	$x(t)\sin(\omega_0 t)\leftrightarrow \frac{j}{2}\left[X(\omega+\omega_0)-X(\omega-\omega_0)\right]$
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t)\leftrightarrow \frac{1}{2}\left[X(\omega+\omega_0)+X(\omega-\omega_0)\right]$
Differentiation wrt time	$\dfrac{d^n x(t)}{dt^n}\leftrightarrow (j\omega)^n\,X(\omega)\quad n=1,2,\ldots$
Integration	$\int_{-\infty}^{t} x(\lambda)\,d\lambda\leftrightarrow \frac{1}{j\omega}X(\omega)+\pi X(0)\,\delta(\omega)$
Convolution in time-domain	$x(t)*v(t)\leftrightarrow X(\omega)V(\omega)$
Multiplication in time-domain	$x(t)v(t)\leftrightarrow \frac{1}{2\pi}X(\omega)*V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)\,v(t)\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\overline{X(\omega)}\,V(\omega)\,d\omega$
Special case of Parseval's theorem	$\int_{-\infty}^{\infty} x^2(t)\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2\,d\omega$
Duality	$X(t)\leftrightarrow 2\pi x(-\omega)$

Continuous-Time Fourier Transform pairs [i.e., $x(t)\leftrightarrow X(\omega)$]

$1,\quad -\infty<t<\infty\leftrightarrow 2\pi\delta(\omega)$

$-0.5+u(t)\leftrightarrow \dfrac{1}{j\omega}$

$u(t)\leftrightarrow \pi\,\delta(\omega)+\dfrac{1}{j\omega}$

$\delta(t)\leftrightarrow 1$

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$e^{-bt}u(t)\leftrightarrow \dfrac{1}{j\omega+b},\quad b>0$

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$p_\tau(t)\leftrightarrow \tau\,\mathrm{sinc}\left(\dfrac{\tau\omega}{2\pi}\right)$

$\tau\,\mathrm{sinc}\left(\dfrac{\tau t}{2\pi}\right)\leftrightarrow 2\pi\,p_\tau(\omega)$

$\Lambda_\tau(t)=\left(1-\dfrac{2\lvert t\rvert}{\tau}\right)p_\tau(t)\leftrightarrow \dfrac{\tau}{2}\mathrm{sinc}^2\left(\dfrac{\tau\omega}{4\pi}\right)$

$\dfrac{\tau}{2}\mathrm{sinc}^2\left(\dfrac{\tau t}{4\pi}\right)\leftrightarrow 2\pi\left(1-\dfrac{2\lvert\omega\rvert}{\tau}\right)p_\tau(\omega)=2\pi\,\Lambda_\tau(\omega)$

$\cos(\omega_0 t)\leftrightarrow \pi\left[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\right]$

$\cos(\omega_0 t+\theta)\leftrightarrow \pi\left[e^{-j\theta}\delta(\omega+\omega_0)+e^{j\theta}\delta(\omega-\omega_0)\right]$

$\sin(\omega_0 t)\leftrightarrow j\pi\left[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)\right]$

$\sin(\omega_0 t+\theta)\leftrightarrow j\pi\left[e^{-j\theta}\delta(\omega+\omega_0)-e^{j\theta}\delta(\omega-\omega_0)\right]$