EE 313 Signals and Systems (Fall 2024)
Project 2 Fourier Analysis of Systems- AC/DC Rectification, Part B

Introduction

<snip> selected between <snip> half-wave rectifying <snip> and **full-wave rectifying circuit** <snip>

In Part B of this project, we will explore how to make our choice of rectifying circuit suitable for a DC power supply. An inexpensive and effective option to implement a DC power supply is to add a capacitor C in parallel with the load resistance R_L <snip> the capacitor acts as a lowpass filter <snip>

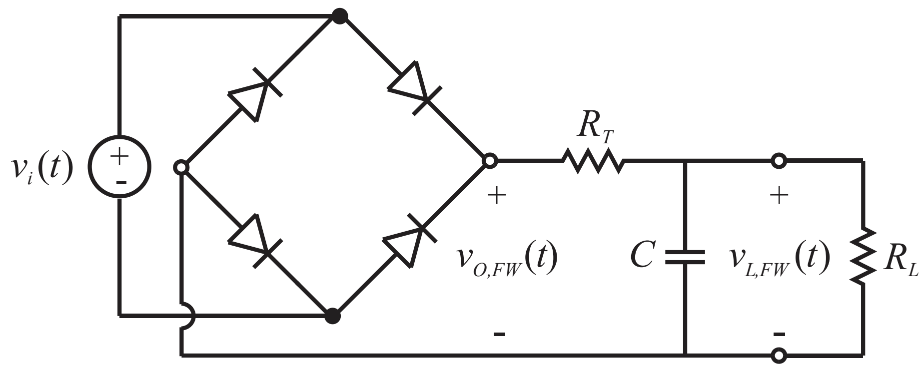


Figure 2 Modified full-wave rectifying circuit with lowpass filtering

Project

Per part A of the project, $v_i(t) = V_m \cos(\omega t)$ w/ V_m = 176 V, f = 400 Hz, $T = 1/f$, $R_T = 4\,\Omega$, and $R_L = 4\,\Omega$.

1) Based on the type of rectification selected in part A of the project, select <snip> Figure 2 (***xx* = *FW***), Then, complete the following:

   a) Find the general <snip> frequency domain transfer function <snip>, i.e., $H_3(\omega) = \dfrac{V_{L,xx}(\omega)}{V_{O,xx}(\omega)}$.

   The relation between $V_{L,FW}$ and $V_{O,FW}$ is found by simple voltage division in the frequency-domain

$$V_{L,FW}(\omega) = \frac{\left(\frac{1}{j\omega C} \middle\| R_L\right)}{R_T + \left(\frac{1}{j\omega C} \middle\| R_L\right)} V_{O,FW}(\omega).$$

   After some simplification, the transfer function is found to be:

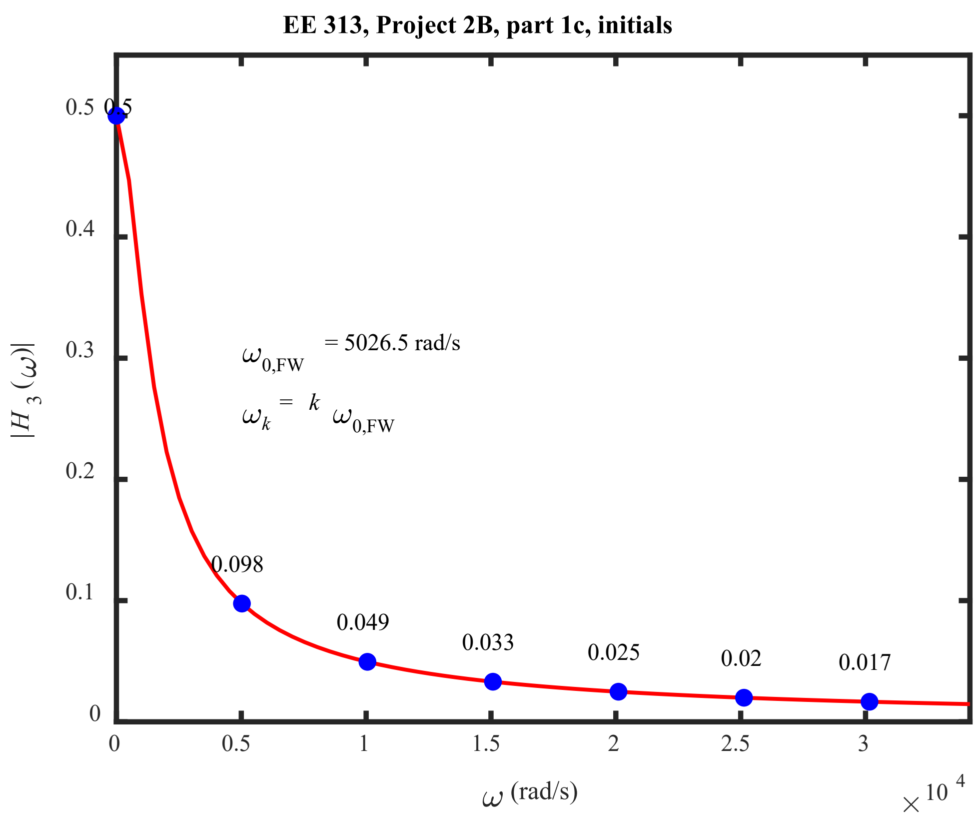
$$H_3(\omega) = \frac{V_{L,FW}(\omega)}{V_{O,FW}(\omega)} = \frac{R_L}{(R_L + R_T) + j\omega R_L R_T C} = \frac{1}{(1 + R_T / R_L) + j\omega R_T C} \qquad -\infty < \omega < \infty.$$

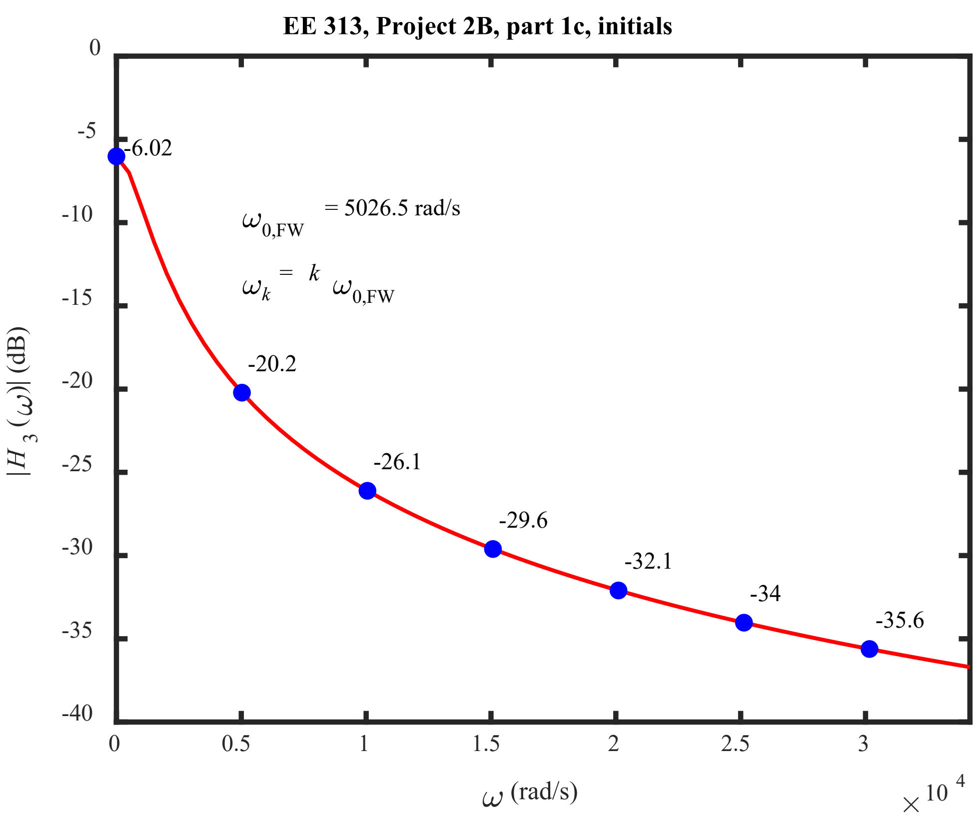
   b) Assuming C = 500 μF and the given resistor values, find the specific transfer function $H_3(\omega)$.

$$H_3(\omega) = \frac{R_L}{(R_L + R_T) + j\omega R_L R_T C} = \frac{4}{(4+4) + j\omega 4(4)500 \cdot 10^{-6}}$$

$$H_3(\omega) = \frac{4}{8 + j\omega 0.008} = \frac{1}{2 + j\omega 0.002} = \frac{0.5}{1 + j\omega 0.001} \qquad -\infty < \omega < \infty$$

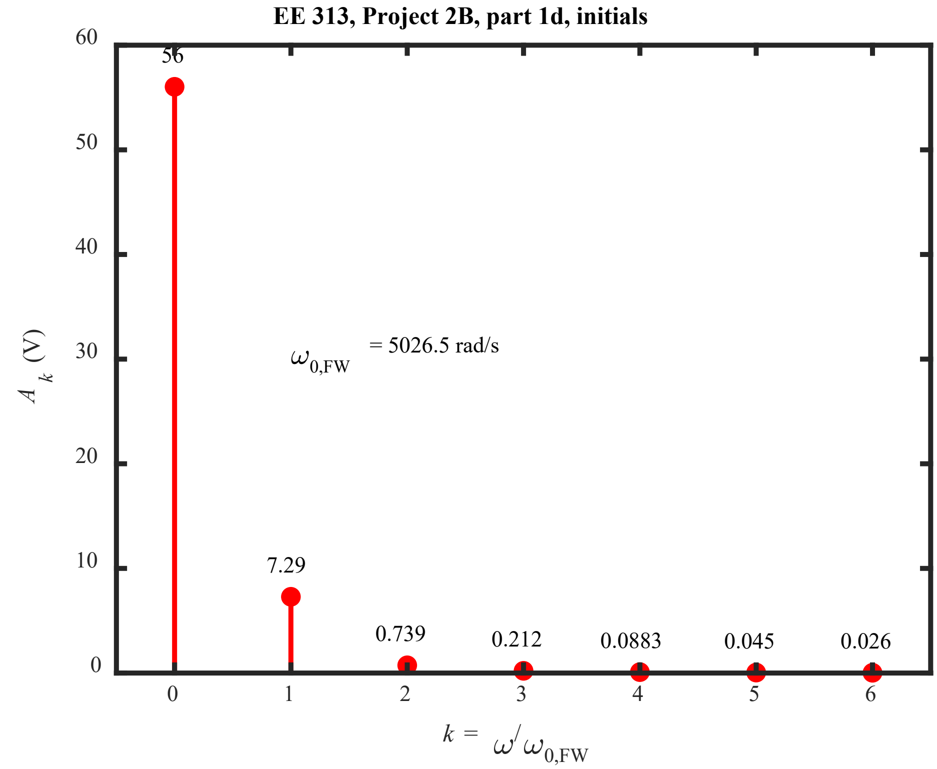
c) <snip> plot <snip> $|H_3(\omega)|$ (unitless, 0 to 0.55 vertical scale) and $|H_3(\omega)|$ (dB, 0 to -40 vertical scale) for $0 \le \omega \le 6.8\omega_{0,\mathbf{FW}}$ (2 plots). Also, evaluate <snip> $H_3(\omega_k)$ for $0 \le k \le 6$, and place **labeled** markers (dots) corresponding to $|H_3(\omega_k)|$ on the same plots. Include m-file.

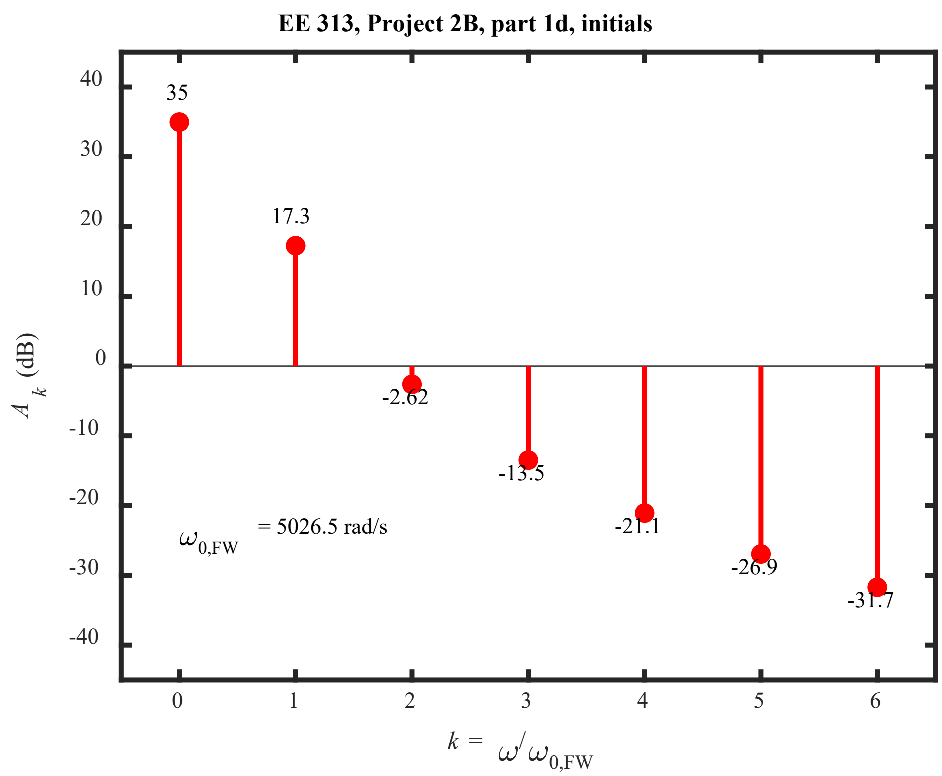




<snip>

d) <snip> create two stem plots of the <u>filtered</u> **amplitude/A_k** (vertical scale of 0 to 60 V) line spectra for $v_{L,xx}(t)$ versus <snip> ω_k, i.e., $0 \le k \le 6$ with horizontal scale ranging from -0.5 to 6.5 <snip> Next, create a stem plot of the **amplitude/A_k** line spectra in decibels ($-45 \le \text{dB} \le 45$) with same horizontal scale. For all plots, label all stems. Include m-file(s).





<snip>

e) Estimate the total power $P_{L,FW}$ in the filtered load voltage by adding the power in the dc plus first ten harmonics. For the three largest line spectra, fill-in the table below. What fraction of the total power is contained in these three?

Table 1 Power in three largest line spectra for filtered full-wave rectification

harmonic #	***f* (Hz)**	**amplitude (V)**	***P* (W)**	**% of $P_{L,xx}$**
0	0	56.02254	784.631246	99.151621
1	800	7.287407	6.638287	0.8388615
2	1600	0.739373	0.068334	0.008635

- **$P_{L,FW} \cong$ 791.344849 W** of which the first three account for **99.999118%.**

<snip>

2) Next, we will examine the time-domain filtered load voltage $v_{L,xx}(t)$ assuming $C = 500$ μF and the given resistor values.

a) Type out the equation for the trigonometric Fourier series of $v_{L,xx}(t)$. **Instructions:** Separate the dc term from the rest of the summation. For simplicity, you may use $|H_3(k\omega_0)|$ and $\angle H_3(k\omega_0)$ in the summation part (i.e., $k = 1, 2, \ldots$) of the expression for $v_{L,xx}(t)$ as long as you reference the equation for $H_3(\omega)$.

We have the Fourier series coefficients for the unfiltered $v_{L,FW}(t)$ in Table 3 of part A. The Fourier series coefficients for $v_{O,FW}(t)$ are simply those for the unfiltered $v_{L,FW}(t)$ multiplied by 2 (i.e., divide out $H_2 = 0.5$).

Table 2 Fourier series coefficients for $v_{O,FW}(t)$

Coefficient	**Value/expression**
$a_{0,FW}$	$2V_m / \pi = 112.04508$ V
$a_{k,FW}$ $(k = 1, 2, \ldots)$	$\dfrac{-4V_m\cos(\pi k)}{\pi(4k^2-1)} = \dfrac{-224.09016\cos(\pi k)}{4k^2-1}$ (V)
$b_{k,FW}$ $(k = 1, 2, \ldots)$	0

We get the Fourier series for $v_{L,FW}(t)$ by multiplying the magnitude of each term of $v_{O,FW}(t)$ by $|H_3(k\omega_{0,FW})|$ and adding the phase angle of $H_3(k\omega_{0,FW})$ to the argument of each $\cos(k\omega_{0,FW}t)$ term, i.e., $\cos[k\omega_{0,FW}t + \angle H_3(k\omega_{0,FW})]$.

$$v_{L,FW}(t) = |H_3(0)|a_{0,FW} + \sum_{k=1}^{\infty} |H_3(k\omega_{0,FW})| a_{k,FW} \cos\left(k\omega_{0,FW}t + \angle H_3(k\omega_{0,FW})\right)$$
$$= (0.5)\frac{2V_m}{\pi} + \sum_{k=1}^{\infty} |H_3(k\omega_{0,FW})| \left(\frac{-4V_m \cos(\pi k)}{\pi(k^2-1)}\right)\cos\left(k\omega_{0,FW}t + \angle H_3(k\omega_{0,FW})\right)$$
$$= (0.5)\frac{(2)176}{\pi} + \sum_{k=1}^{\infty} |H_3(k1600\pi)| \left(\frac{-4(176)\cos(\pi k)}{\pi(k^2-1)}\right)\cos\left(k1600\pi t + \angle H_3(k1600\pi)\right)$$
$$v_{L,FW}(t) = 56.0225 + \sum_{k=1}^{\infty} |H_3(k1600\pi)| \left(\frac{-224.09\cos(\pi k)}{k^2-1}\right)\cos\left(k1600\pi t + \angle H_3(k1600\pi)\right) \text{ (V)} \quad -\infty < t < \infty$$

where $\omega_{0,FW} = 2\pi f_{0,FW} = 1600\pi = 5026.548$ rad/s, and

$$H_3\left(k\omega_{0,FW}\right) = H_3\left(k1600\pi\right) = \frac{4}{8 + jk1600\pi\, 0.008} = \frac{0.5}{1 + j1.6\pi k} \qquad -\infty < \omega < \infty .$$

b) Calculate $a_{0,xx}$, $a_{1,xx}$, and $a_{2,xx}$. Calculate $H(0)$, $H(\omega_{0,xx})$, and $H(2\omega_{0,xx})$ [put in polar form w/ angle in degrees]. Type out the equation for a truncated trigonometric Fourier series of $v_{L,xx}(t)$ that includes the dc term plus first **two** harmonics with all terms enumerated.

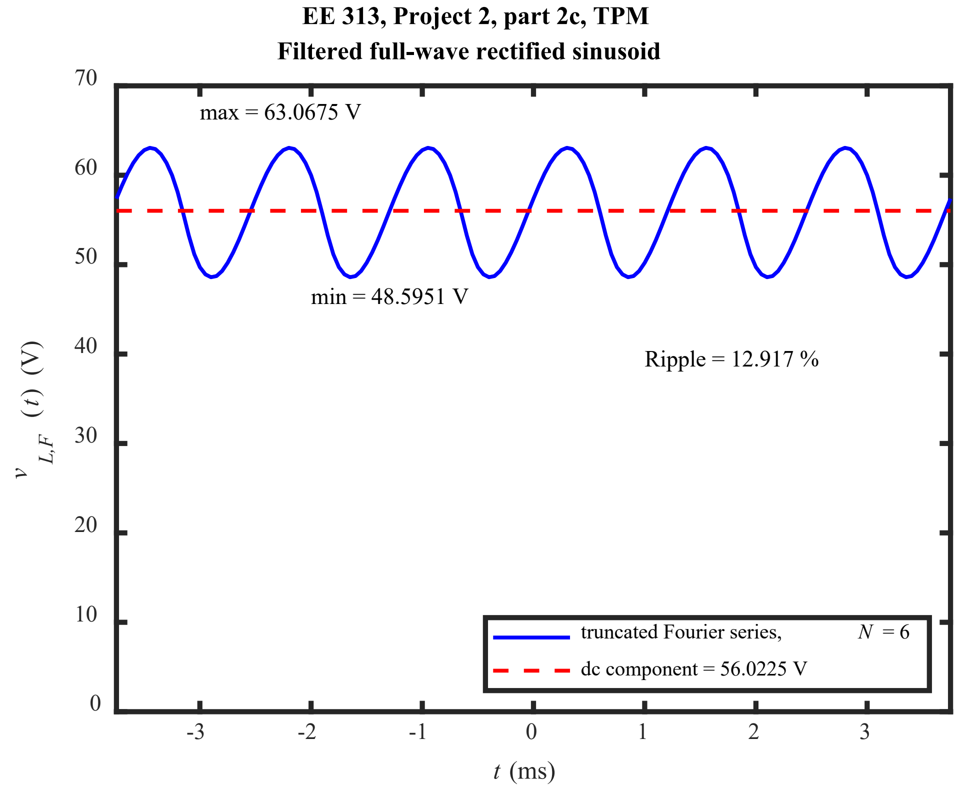
$$H_3(0) = \frac{0.5}{1 + j0} = 0.5 \qquad \& \qquad a_{0,FW} = 2(176)/\pi = 112.04508 \text{ V},$$

$$H_3(1600\pi) = \frac{0.5}{1 + j1.6\pi} = 0.09756\angle -78.748° \;\; \& \;\; a_{1,FW} = \frac{-4(176)\cos(\pi)}{\pi(4(1)^2-1)} = 74.6967 \text{ (V)}$$

$$H_3(3200\pi) = \frac{0.5}{1 + j1.6\pi 2} = 0.04949\angle -84.319° \;\; \& \;\; a_{2,FW} = \frac{-4(176)\cos(\pi 2)}{\pi(4(2)^2-1)} = -14.9393 \text{ (V)}$$

$$v_{L,FW}(t) = (0.5)112.045 + (0.09756)74.7\cos\left(\omega_{0,FW}t - 78.748°\right) - (0.04949)14.9393\cos\left(2\omega_{0,FW}t - 84.319°\right)$$
$$= 56.0225 + 7.2874\cos\left(1600\pi t - 78.748°\right) - 0.7393\cos\left(3200\pi t - 84.309°\right) \quad \text{(V)}$$
$$= 56.0225 + 7.2874\cos\left(5026.55t - 1.3744\right) - 0.7393\cos\left(10053.1t - 1.47165\right) \quad \text{(V)} \quad -\infty < t < \infty$$

c) Plot the truncated Fourier series of $v_{L,xx}(t)$ using the dc term plus the first six harmonics. Put a labeled horizontal dashed line at the value of the dc term $(v_{L,xx})_{dc}$. For horizontal and vertical scales, use $-1.5T \le t \le 1.5T$ and $0 \le V \le 70$ V, respectively. Determine the maximum $(v_{L,xx})_{max}$ and minimum $(v_{L,xx})_{min}$ values of $v_{L,xx}(t)$ and put on the plot. While ideally the filtered output would be equal to $(v_{L,xx})_{dc}$, the output quality of practical dc power supplies is characterized by their ripple R defined as $R = \left[\left(\left(v_{L,xx}\right)_{max} - \left(v_{L,xx}\right)_{min}\right)/2\right] \Big/ \left(v_{L,xx}\right)_{dc} * 100\%$. Find the ripple R for this circuit.



➢ **$(v_{L,FW})_{max}$ = 63.0675 V** and **$(v_{L,FW})_{min}$ = 48.5951 V**

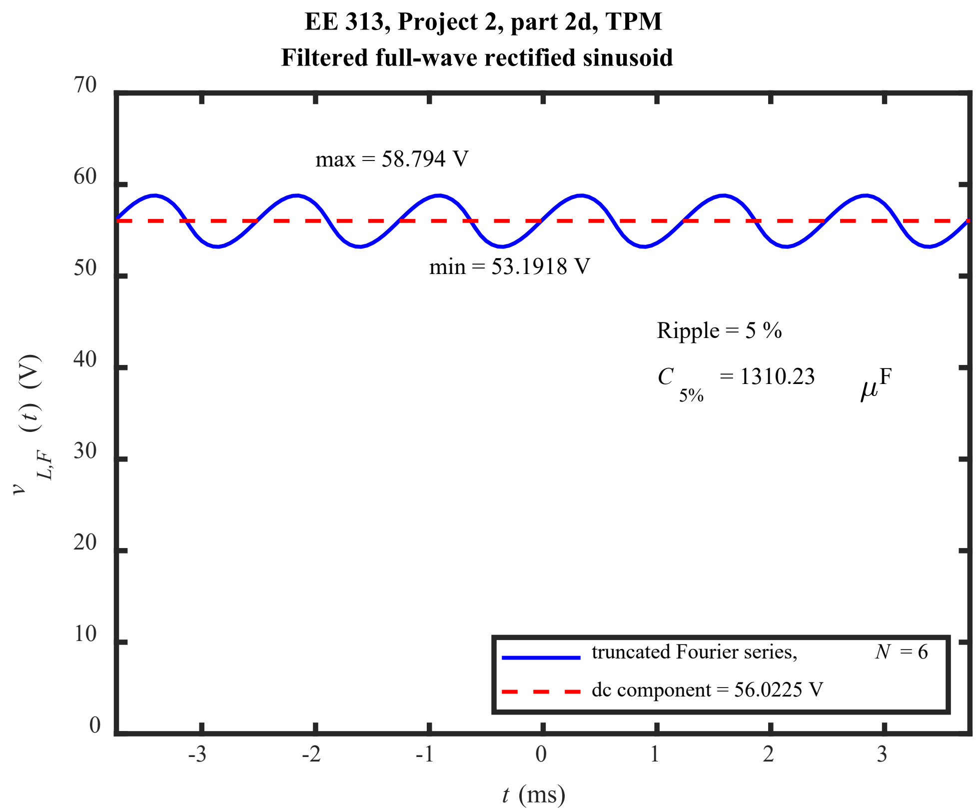
➢ $$R = \left(\frac{\left[(v_{L,F})_{max} - (v_{L,F})_{min} \right] / 2}{(v_{L,F})_{dc}} \right) 100\% = \left(\frac{[63.0675 - 48.5951]/2}{56.0225} \right) 100\% \Rightarrow \mathbf{R = 12.917\%}$$

<snip>

3) <snip> modify our circuit by changing the capacitor value C (other parts of circuit are unchanged). As a 'rule of thumb', a 5% ripple is considered acceptable for most dc power supply applications.

a) <snip> determine <snip> $C_{5\%}$ (μF) needed to achieve a 5% ripple. Plot <snip> $v_{L,xx}(t)$ <snip> Put a labeled horizontal dashed line <snip> $(v_{L,xx})_{dc}$. For horizontal & vertical scales, use $-1.5T \le t \le 1.5T$ & $0 \le V \le 60$ V<snip> Determine <snip> $(v_{L,xx})_{max}$ & <snip> $(v_{L,xx})_{min}$ values of $v_{L,xx}(t)$ & put on plot.

➢ **$C_{5\%}$ = 1310.23 μF**



<snip>

b) Using $C_{5\%}$, estimate the total power $P_{L,xx}$ in the filtered load voltage by adding the power in the dc plus first ten harmonics. For the three largest line spectra, fill-in the table below. What fraction of the total power is contained in these three?

Table 3 Power in three largest line spectra for 5% filtered full-wave rectification

harmonic #	f (Hz)	amplitude (V)	P (W)	% of $P_{L,xx}$
0	0	56.02254	784.631246	99.871407
1	800	2.827328	0.999223	0.127186
2	1600	0.283342	0.010035	0.001277

- ➢ **$P_{L,FW,5\%} \cong$ 785.6415244 W** of which the first three account for **99.99987%.**

<snip>