EE 313 Signals and Systems (Fall 2024)
Project 2 Fourier Analysis of Systems- AC/DC Rectification, Part B

Introduction

In this project, we will examine rectifying circuits in light of our study of Fourier series and the Fourier transform. Signal rectification, both full-wave and half-wave, is the primary means by which AC power/signals are converted to DC power/signals. In Part A of this project, we selected between the modified half-wave rectifying circuit (Figure 2 of part A) and full-wave rectifying circuit (Figure 4 of part A) as being most suitable for a DC power supply in terms of the Fourier series.

In Part B of this project, we will explore how to make our choice of rectifying circuit suitable for a DC power supply. An inexpensive and effective option to implement a DC power supply is to add a capacitor C in parallel with the load resistance R_L as shown in Figures 1 and 2. In the frequency domain, the capacitor acts as a lowpass filter, i.e., reduces the amplitude of the harmonics $\omega_k = k\omega_0$ where $k > 0$.

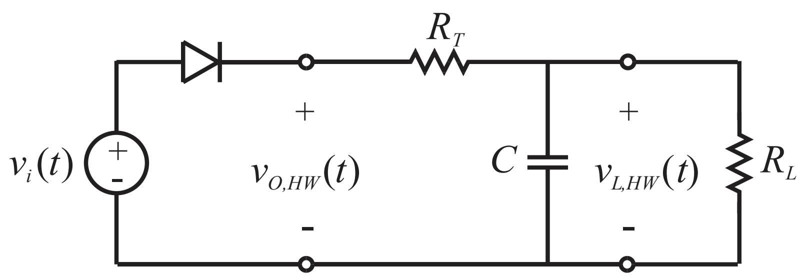

Figure 1 Modified half-wave rectifying circuit with lowpass filtering

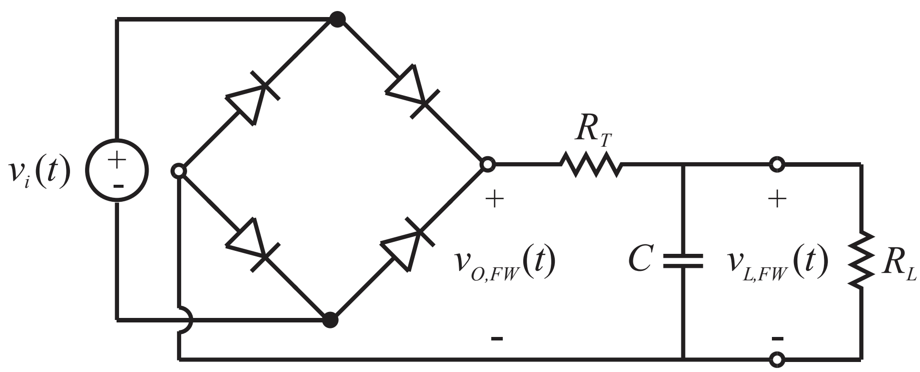

Figure 2 Modified full-wave rectifying circuit with lowpass filtering

Project

Per part A of the project, $v_i(t) = V_m \cos(\omega t)$ w/ V_m = 176 V, f = 400 Hz, R_T = 4 Ω, and R_L = 4 Ω.

1) Based on the type of rectification selected in part A of the project, select either the circuit shown in Figure 1 (set $xx = HW$) or Figure 2 (set $xx = FW$). Then, complete the following:
   a) Find the general (leave all circuit components as variables) frequency domain transfer function $H_3(\omega)$ that relates the filtered phasor load voltage to the phasor output voltage of the rectifying circuit, i.e., $H_3(\omega) = \dfrac{V_{L,xx}(\omega)}{V_{O,xx}(\omega)}$.

Hint: Use a phasor equivalent circuit where $v_{O,xx}(t)$ is a sinusoid source at some frequency ω to find $V_{L,xx}(\omega)$ in terms of $V_{O,xx}(\omega)$.

b) Assuming C = 500 μF and the given resistor values, find the specific transfer function $H_3(\omega)$.

c) Using MATLAB, plot the magnitude $|H_3(\omega)|$ (unitless, 0 to 0.55 vertical scale) and $|H_3(\omega)|$ (dB, 0 to -40 vertical scale) for $0 \le \omega \le 6.8\omega_{0,xx}$ (2 plots). Also, evaluate the transfer function at the harmonic frequencies $\omega_k = k\omega_{0,xx}$ up to the sixth, i.e., $H_3(\omega_k)$ for $0 \le k \le 6$, and place **labeled** markers (dots) corresponding to $|H_3(\omega_k)|$ on the same plots. Include m-file.

d) Using MATLAB, create two stem plots of the filtered **amplitude/A_k** (vertical scale of 0 to 60 V) line spectra for $v_{L,xx}(t)$ versus $k = \omega_{k,xx} / \omega_0$ up to the sixth harmonic frequency ω_k, i.e., $0 \le k \le 6$ with horizontal scale ranging from -0.5 to 6.5 to avoid obscuring stems/labels. Next, create a stem plot of the **amplitude/A_k** line spectra in decibels ($-45 \le$ dB ≤ 45) with same horizontal scale. For all plots, label all stems. Include m-file(s).

Hint: Recall how the amplitude and phase of sinusoids are modified by transfer functions.

e) Estimate the total power $P_{L,xx}$ in the filtered load voltage by adding the power in the dc plus first ten harmonics. For the three largest line spectra, fill-in the table below. What fraction of the total power is contained in these three?

harmonic #	f (Hz)	amplitude (V)	P (W)	% of $P_{L,xx}$

2) Next, we will examine the time-domain filtered load voltage $v_{L,xx}(t)$ assuming C = 500 μF and the given resistor values.

a) Type out the equation for the trigonometric Fourier series of $v_{L,xx}(t)$. **Instructions:** Separate the dc term from the rest of the summation. For simplicity, you may use $\left|H_3\left(k\omega_{0,xx}\right)\right|$ and $\angle H_3\left(k\omega_{0,xx}\right)$ in the summation part (i.e., k = 1, 2, …) of the expression for $v_{L,xx}(t)$ as long as you reference the equation for $H_3(\omega)$.

b) Calculate $a_{0,xx}$, $a_{1,xx}$, and $a_{2,xx}$. Calculate $H(0)$, $H(\omega_{0,xx})$, and $H(2\omega_{0,xx})$ [put in polar form w/ angle in degrees]. Type out the equation for a truncated trigonometric Fourier series of $v_{L,xx}(t)$ that includes the dc term plus first **two** harmonics with all terms enumerated.

c) Plot the truncated Fourier series of $v_{L,xx}(t)$ using the dc term plus the first six harmonics. Put a labeled horizontal dashed line at the value of the dc term $(v_{L,xx})_{dc}$. For horizontal and vertical scales, use $-1.5T \le t \le 1.5T$ and $0 \le V \le 70$ V, respectively. Determine the maximum $(v_{L,xx})_{max}$ and minimum $(v_{L,xx})_{min}$ values of $v_{L,xx}(t)$ and put on the plot. While ideally the filtered output would be equal to $(v_{L,xx})_{dc}$, the output quality of practical dc power supplies is characterized by their ripple R defined as $R = \left[\left(\left(v_{L,xx}\right)_{max} - \left(v_{L,xx}\right)_{min}\right)/2\right] \Big/ \left(v_{L,xx}\right)_{dc} * 100\%$. Find the ripple R for this circuit.

3) Next, we will modify our circuit by changing the capacitor value C (other parts of circuit are unchanged). As a 'rule of thumb', a 5% ripple is considered acceptable for most dc power supply applications.

   a) By trial-and-error, determine the capacitor value $C_{5\%}$ (μF) needed to achieve a 5% ripple. Plot the associated truncated Fourier series of $v_{L,xx}(t)$ using the dc term plus the first six harmonics. Put a labeled horizontal dashed line at the value of the dc term $(v_{L,xx})_{\mathrm{dc}}$. For horizontal and vertical scales, use $-1.5T \leq t \leq 1.5T$ and $0 \leq V \leq 60$ V, respectively. Determine the maximum $(v_{L,xx})_{\max}$ and minimum $(v_{L,xx})_{\min}$ values of $v_{L,xx}(t)$ and put on the plot.

   b) Using $C_{5\%}$, estimate the total power $P_{L,xx}$ in the filtered load voltage by adding the power in the dc plus first ten harmonics. For the three largest line spectra, fill-in the table below. What fraction of the total power is contained in these three?

harmonic #	***f*** **(Hz)**	**amplitude (V)**	***P*** **(W)**	**% of $P_{L,xx}$**

Report Instructions

You may work singly or with a partner. The results should be organized into a word-processed short report.

- Include: 1) cover page, 2) Introduction, 3) body (broken down into subsections based on the steps in project), and 4) Summary & Conclusions.
- Put the calculations, results, m-files, and plots/figures **in the body** of the report in the order specified as they occur. Appendices are **NOT** to be used as a "dumping ground" for figures and m-files. However, **longer** mathematical derivations may be attached as Appendices **if referenced in the text** of the report.
- All plots/figures/tables should be numbered and captioned to allow easy reference.
- In addition to HW requirements given in the syllabus, use a font size ≥ 12 points & line spacing ≥ 1.1.
- Numerical results that are specifically requested should be put on separate line(s), not 'buried' in the middle of a paragraph.
- To enhance readability, figures/plots should span width of page and face either the bottom or right of page. Also, remember that text on figures/plots that is too small to read might as well not exist.
- On all plots, label horizontal and vertical axes, and insert a horizontal axis at 0. Put "EE 313, Project # & part #, *your initials*, date" in the title. If a plot contains more than one trace, use different line colors/types and a legend to clearly identify each trace.
- For all m-files, put filename, EE 313, Project # & part #, *your name*, and date in comment lines.
- Staple results together and turn-in the project report on **<u>Monday, November 25, 2024</u>**.