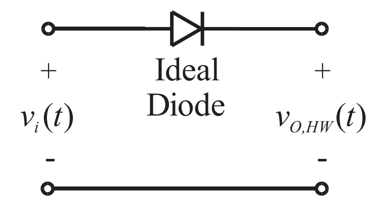
EE 313 Signals and Systems (Fall 2024)
Project 2 Fourier Analysis of Systems- AC/DC Rectification, Part A

Introduction

In this project, we will examine rectifying circuits in light of our study of Fourier series and the Fourier transform. Signal rectification, both full-wave and half-wave, is the primary means by which AC power/signals are converted to DC power/signals. In Part A of this project, we will compare the full-wave and half-wave rectification of a sinusoid in terms of the Fourier series.

Background



For an ideal diode (right) with input voltage $v_i(t) = V_m \cos(\omega t)$, the output voltage $v_{O,HW}(t) = \begin{cases} V_m \cos(\omega t) & v_i(t) \geq 0 \\ 0 & v_i(t) < 0 \end{cases}$.

A potential **half-wave rectifying** circuit is shown in Figure 1. In this circuit, a Thevenin equivalent circuit model represents the source, $v_S(t)$ is the output of the source, and $v_L(t)$ is the load voltage.

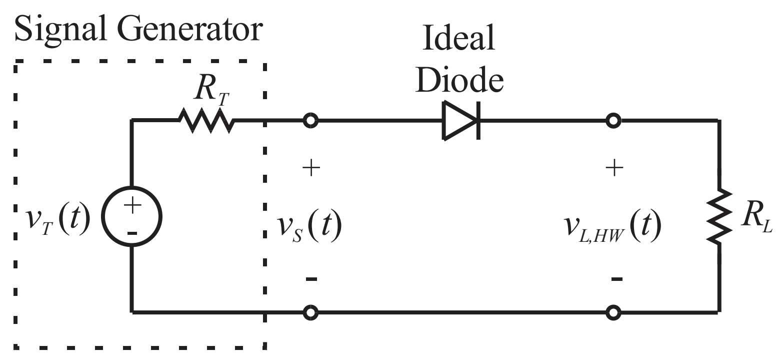


Figure 1 Half-wave rectifying circuit

To simplify the analysis, swap the positions of the ideal diode and Thevenin equivalent source resistance R_T in the circuit. Letting $v_T(t) = v_i(t)$, we get the circuit shown in Figure 2.

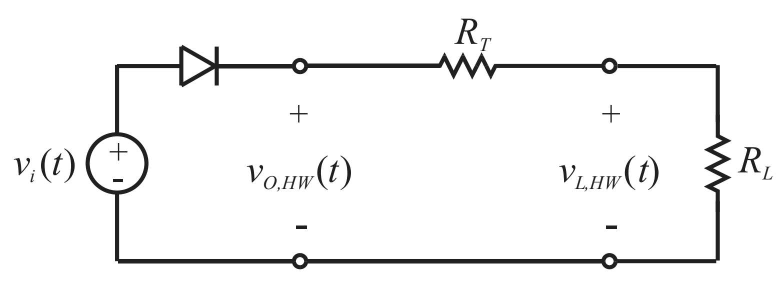
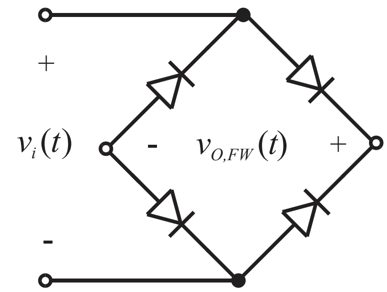


Figure 2 Modified half-wave rectifying circuit

By voltage division, the load voltage is $v_{L,HW}(t) = \left(\frac{R_L}{R_T + R_L} \right) v_{O,HW}(t)$ with a corresponding transfer function $H_1(\omega) = \frac{V_{L,HW}(\omega)}{V_{O,HW}(\omega)} = \frac{R_L}{R_T + R_L}$.



For an ideal diode bridge rectifier (right) with input $v_i(t) = V_m \cos(\omega t)$, the output voltage is $v_{O,FW}(t) = |V_m \cos(\omega t)|$.

A potential **full-wave rectifying** circuit, using a diode bridge rectifier, is shown in Figure 3. To simplify the analysis, swap the positions of the ideal diode bridge rectifier and Thevenin equivalent source resistance R_T in the circuit. Letting $v_T(t) = v_i(t)$, we get the circuit shown in Figure 4.

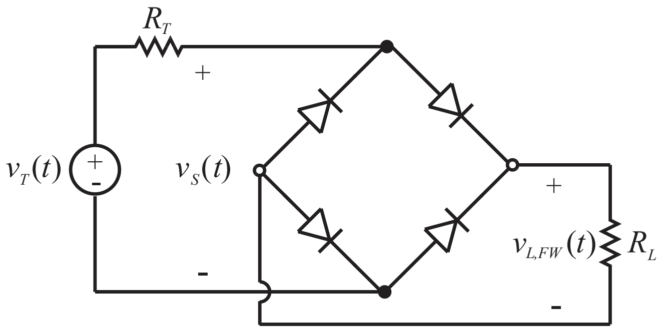


Figure 3 Full-wave rectifying circuit

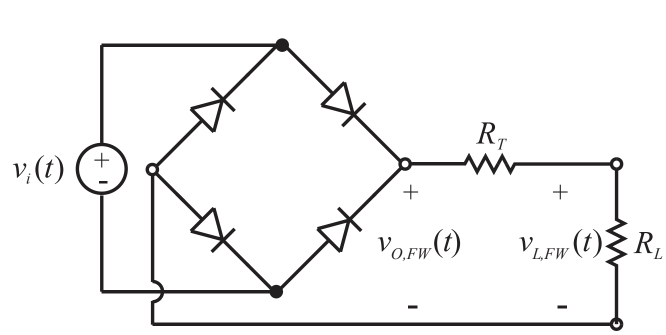


Figure 4 Modified full-wave rectifying circuit

By voltage division, the load voltage is $v_{L,FW}(t) = \left(\dfrac{R_L}{R_T + R_L}\right) v_{O,FW}(t)$ with a corresponding transfer function $H_2(\omega) = \dfrac{V_{L,FW}(\omega)}{V_{O,FW}(\omega)} = \dfrac{R_L}{R_T + R_L} = H_1(\omega)$.

Project

1) For the half-wave rectifying circuit (Figure 2) with $V_m = 176$ V, $f = 400$ Hz, $R_T = 4\ \Omega$, and $R_L = 4\ \Omega$:
   a) Find $H_1(\omega)$ and use MATLAB to graph the exact expression for $v_{L,HW}(t)$. For the horizontal and vertical scales, use $-1.5T \le t \le 1.5T$ ($T = 1/f$) and $-10 \le V \le 120$ V, respectively. Include m-file.
   b) Find the period T_{HW} and fundamental frequency $\omega_{0,HW}$ of $v_{L,HW}(t)$. Then, calculate the coefficients for the trigonometric Fourier series of $v_{L,HW}(t)$. In a table, list simplified expressions for the coefficients $a_{0,HW}$, $a_{1,HW}$, & $b_{1,HW}$ as well as $a_{k,HW}$ & $b_{k,HW}$ ($k = 2, 3, \ldots$). Then, type-out the equation for the trigonometric Fourier series of $v_{L,HW}(t)$ using the form given by equation (3.4) of the text. [Hints: See equations (3.5), (3.6), and (3.7). The $k = 1$ coefficients are separated for a reason.]

Coefficient	Value/expression
$a_{0,HW}$	
$a_{1,HW}$	
$b_{1,HW}$	
$a_{k,HW}$ ($k = 2, 3, \ldots$)	
$b_{k,HW}$ ($k = 2, 3, \ldots$)	

c) To verify results of part b), plot the truncated trigonometric Fourier series (dashed line) and the exact expression (solid line) of $v_{L,HW}(t)$ using MATLAB for two cases (2 graphs). For the first truncated trigonometric Fourier series, use the dc term plus the first two harmonics. Next, use the dc term plus the first six harmonics. Use $-1.5T \le t \le 1.5T$ and $-10 \le V \le 120$ V for horizontal and vertical scales, respectively. Include m-file.

d) Using MATLAB, create a stem plot of the **amplitude** (vertical scale being 0 to 60 V) line spectra for $v_{L,HW}(t)$ versus $k = \omega_k / \omega_0$ up to the tenth harmonic frequency ω_k, i.e., $0 \le k \le 10$ with horizontal scale ranging from -0.7 to 10.8 to avoid obscuring stems/labels. Next, create a stem plot of the **amplitude line spectra** in decibels ($-10 \le \text{dB} \le 40$) with same horizontal scale. For this plot, round values less than minimum vertical scale to minimum vertical scale. For both plots, label all *finite* stems. Include m-file(s). [Hint: $A_k(\text{dB}) = 20\log_{10}(A_k)$.]

e) Analytically find the total average power $P_{L,HW}$ of $v_{L,HW}(t)$ in Watts. Then, find the power and fraction (%) of $P_{L,HW}$ is contained in each of the three largest line spectra. For the three largest line spectra, fill-in the table below. What fraction of the total power is contained in these three? [Hints: The DC line spectra is harmonic 0 (or 'DC'). Parseval's Theorem. To get the average power P in Watts, let $x(t) = v_L(t)$ and divide by R_L in both (3.28) and (3.29).]

harmonic #	***f* (Hz)**	**amplitude (V)**	***P* (W)**	**% of $P_{L,HW}$**

2) For the full-wave rectifying circuit (Figure 4) with $V_m = 176$ V, $f = 400$ Hz, $R_T = 4\ \Omega$, and $R_L = 4\ \Omega$:

a) Find $H_2(\omega)$ and use MATLAB to graph the exact expression for $v_{L,FW}(t)$. For the horizontal and vertical scales, use $-1.5T \le t \le 1.5T$ ($T = 1/f$) and $-10 \le V \le 120$ V, respectively. Include m-file.

b) Find the period T_{FW} and fundamental frequency $\omega_{0,FW}$ of $v_{L,FW}(t)$. Then, calculate the coefficients for the trigonometric Fourier series of $v_{L,FW}(t)$. In a table, list simplified expressions for the coefficients $a_{0,FW}$ as well as $a_{k,FW}$ & $b_{k,FW}$ ($k = 1, 2, \ldots$). Then, type-out the equation for the trigonometric Fourier series of $v_{L,FW}(t)$ per equation (3.4) of the text.

Coefficient	**Value/expression**
$a_{0,FW}$	
$a_{k,FW}$ ($k = 1, 2, \ldots$)	
$b_{k,FW}$ ($k = 1, 2, \ldots$)	

c) To verify results of part b), plot the truncated trigonometric Fourier series (dashed line) and the exact expression (solid line) of $v_{L,FW}(t)$ using MATLAB for two cases (2 graphs). For the first truncated trigonometric Fourier series, use the dc term plus the first two harmonics. Next, use the dc term plus the first six harmonics. Use $-1.5T \le t \le 1.5T$ and $-10 \le V \le 120$ V for horizontal and vertical scales, respectively. Include m-file.

d) Using MATLAB, create a stem plot of the **amplitude** (vertical scale being 0 to 60 V) line spectra for $v_{L,FW}(t)$ versus $k = \omega_k / \omega_0$ up to the tenth harmonic frequency ω_k, i.e., $0 \leq k \leq 10$ with horizontal scale ranging from -0.7 to 10.8 to avoid obscuring stems/labels. Next, create a stem plot of the **amplitude line spectra** in decibels ($-10 \leq \text{dB} \leq 40$) with same horizontal scale. For this plot, round values less than minimum vertical scale to minimum vertical scale. For both plots, label all stems. Include m-file(s).

e) Analytically find the total average power $P_{L,FW}$ of $v_{L,FW}(t)$ in Watts. Then, find the power and fraction (%) of $P_{L,FW}$ is contained in each of the three largest line spectra. For the three largest line spectra, fill-in the table below. What fraction of the total power is contained in these three?

harmonic #	***f*** **(Hz)**	**amplitude (V)**	***P*** **(W)**	**% of $P_{L,FW}$**

3) In the Summary and Conclusions, decide whether full-wave or half-wave rectification is most efficient when generating DC power/signals from sinusoids. Clearly state and justify (quantitatively) your answer based on the results of steps 1) and 2).

Report Instructions

You may work **singly or with a partner**. The results should be organized into a word-processed short report.

- Include: 1) cover page, 2) Introduction, 3) body (broken down into subsections based on the steps in project), and 4) Summary & Conclusions.
- Put the calculations, results, m-files, and plots/figures **in the body** of the report in the order specified as they occur. Appendices are **NOT** to be used as a "dumping ground" for figures and m-files. However, **longer** mathematical derivations may be attached as Appendices **if referenced in the text** of the report.
- All plots/figures/tables should be numbered and captioned to allow easy reference.
- In addition to HW requirements (e.g., fronts of pages only) given in the syllabus, use a font size ≥ 12 points & line spacing ≥ 1.1.
- Numerical results that are specifically requested should be put on separate line(s), not 'buried' in the middle of a paragraph.
- To enhance readability, figures/plots should span width of page and face either the bottom or right of page. Also, remember that text on figures/plots that is too small to read might as well not exist.
- On all plots, label horizontal and vertical axes, and insert a horizontal axis at 0. Put "EE 313, Project # & part #, *your initials*, date" in the title. If a plot contains more than one trace, use different line colors/types and a legend to clearly identify each trace.
- For all m-files, put filename, EE 313, Project # & part #, *your name*, and date in comment lines.
- Staple results together and turn-in the project report on **Friday, November 1, 2024**.