EE 313 Signals and Systems (Fall 2024)
Project 1 Unit Pulse Response/Convolution Representation, Part B

Introduction

In this lab, we will examine finding the output of a discrete-time (DT) filter for a given input signal by convolution, i.e., convolve the input signal with the unit pulse response of the filter. In part A of the project, we designed a 6^{th}-order Chebyshev Type I DT IIR LP filter based on an analog cutoff frequency $f_c = 160$ Hz and passband ripple of $R = 0.25$ dB using a sampling period $T_S = 1.5$ ms. This type of filter has an infinite impulse response (IIR). However, to implement the convolution representation, we will truncate the unit pulse response $h[n]$ to be of length N_{max}, i.e., $0 \le n \le N_{max}$ -1. In essence, we will create a finite impulse response (FIR) DT LP filter based on the IIR DT LP filter.

Project

1) Write an m-file to compute the unit pulse response $h[n]$ of the filter for $0 \le n \le N_{max}$ -1. Start with $N_{max} = 101$. The filter has no initial energy, i.e., $y[n < 0] = 0$. Then, perform the following tasks.

   a) Using Matlab, create a stem plot of $h[n]$ with a horizontal scale of $-1 \le n \le N_{max}$ -1 and a vertical scale from -0.3 to 0.5.

   b) Also, create a stem plot of $h[n]$ with a horizontal scale of $-1 \le n \le 20$ and the same vertical scale. Find and label (value and index) the largest positive $h_{pos}[n_{pos}]$ and negative $h_{neg}[n_{neg}]$ swings in the unit pulse response on stem plot. At a minimum, labels should give the numerical value of stem with at least 3 significant figures. From the plot determine the maximum swing in the unit pulse response $h_{max} = \text{maximum}\{h_{pos}[n_{pos}], |h_{neg}[n_{neg}]|\}$.

   c) Next, find $h_{1\%}[n_{1\%}]$, i.e., the point where the magnitude of the unit pulse response $|h[n]|$ becomes (and stays) **less than** 1% of h_{max}, i.e., $0.01h_{max}$, giving both value and index. Also, give the value and index $h[n_{1\%} - 1]$, i.e., the last point in the unit pulse response which **exceeds** 1% of h_{max}. If necessary, increase N_{max}.

      Hint: Create another figure to your m-file, using the plot() command, showing $h[n]$ as dots with dashed horizontal lines at $\pm 0.01h_{max}$. Set the vertical scale to $\pm 0.02h_{max}$ and change the horizontal scale as needed to 'zoom' in on $h_{1\%}[n_{1\%}]$ and $h[n_{1\%} - 1]$.

   d) Is $N_{max} = 101$ sufficient to find $h_{1\%}[n_{1\%}]$? If so, use this value to create the FIR DT filter. If not, increase N_{max} by steps of 20 (e.g., 121, 141, …) until $h_{1\%}[n_{1\%}]$ is found. Give value of N_{max} used.

   e) Using the stem plot of $h[n]$, is the filter stable? Why?

   f) Give a listing of the m-file.

   Hint: To find $h[n]$, you can use the 'recur' command. Remember to remove coefficient $a_0 = 1$ from the coefficient vector a generated by cheby1() in order to match the recur() format. For example, add the line 'a(1) = [];' to your m-file after the cheby1() command. Or, the commands 'dimpulse' and 'filter' can be used (see MATLAB help).

2) Next, use DT convolution representation to calculate the output from the filter. The DT input signals $v_{in}[n]$ should each be calculated for $0 \le n \le 100$. They are sampled from the continuous time (CT) voltage signal $v_{in}(t) = V_0 \sin(2\pi f t)\, u(t)$ where $V_0 = 100$ V and $f = f_c/3$, f_c, $1.1 f_c$, and $1.25 f_c$ (four separate inputs). The specific results required are:

a) Analytically determine the index n_{acc} and corresponding time $t_{acc} = n_{acc}\,T_S$ where the output signal $v_{out}[n]$ ceases to be accurate (show work).

b) At each frequency $f = f_c/3, f_c, 1.1\,f_c$, and $1.25\,f_c$, find and **tabulate** the expected maximum voltage magnitudes for the output $v_{out}[n]$ after passing through the filter. Format: col. 1 f (Hz), col. 2 V_0 (V), col. 3 $|H(f)|$, and col. 4 $V_0\,|H(f)|$ (V).

Hint: Ideally, the output signal should have a maximum magnitude of $V_0\,|H(f)|$. Use results from step 2f of Project 1, part A.

c) Using the $v_{out}[n]$ data, plot $v_{out}(t)$ (solid line) versus time ($t = n\,T_S$) for the **full length** of the 'conv' output at each of the specified frequencies f (**four** separate plots). Identify/label each plot by the appropriate frequency f. On each of the plots, put a labeled vertical dashed line at time t_{acc} and put labeled horizontal dashed lines at the voltage levels $\pm V_0\,|H(f)|$.

d) Over the accurate time range, examine the magnitude of $v_{out}(t)$ at each of the specified frequencies. Does the DT filter work as expected? Comment on any anomalies, transitions, or other interesting features in the plots. [Prompts: In light of $v_{out}(t)$ shown in the plots, roughly how long Δt_{SS} does it take for the filter to start working or reach a steady-state? After t_{acc}, roughly how long Δt_{stop} does it take for the output to die off? In light Δt_{SS} and Δt_{stop}, roughly how many points would these correspond with in the unit pulse response $h[n]$? Discuss pros and cons of digital versus analog filters.]

e) Give a listing of the m-file.

Hints:

- Use MATLAB 'conv' command to calculate $v_{out}[n] = v_{in}[n] * h[n]$.
- Remember the sampled signal is $v_{\text{in}}[n] = v_{\text{in}}(t_n = nT_S) = V_0 \sin(2\pi f\, nT_S)\, u(nT_S)$.
- The sampling rate T_S and how it 'lines up' with each frequency will determine whether or how often the sampling 'hits' the exact minima and maxima of the signals.

Project report format

The results should be organized into a word-processed short report.

- In addition to syllabus HW format requirements, use font size ≥ 12 points and line spacing ≥ 1.1.
- Include: 1) cover page, 2) Introduction, 3) body (broken down into subsections based on the steps in project), and 4) Summary & Conclusions.
- Put the calculations, results, m-files, and plots/figures **in the body** of the report in the order specified as they occur. Do not use appendices.
- On all plots, label horizontal and vertical axes, and insert a horizontal axis at 0. Put "EE 313, Project # & part #, *your initials*, date" in the title.
- Numerical results that are specifically requested should be put on separate line(s), not 'buried' in the middle of a paragraph.
- To enhance readability, figures/plots should span width of page and face either the bottom or right of page. Also, remember that text on figures/plots that is too small to read might as well not exist.
- For all m-files, put the filename, EE 313, Project # & part #, *your name*, and date in comment lines.
- Staple results together and turn-in the project report on **Wednesday, October 16, 2024**.