EE 313 Signals and Systems (Fall 2024)
Project 1 Unit Pulse Response/Convolution Representation, Part A

<snip>

Project

1) Using <snip> $T_S = 1.5$ ms, write an m-file to design a 6th-order Chebyshev Type I DT IIR LP filter based on an analog <snip> $f_c = 160$ Hz and passband ripple of $R = 0.25$ dB. The specific results desired are:

   a) Calculate and list the corresponding DT (Ω) and normalized (Wp) DT cutoff frequencies. Is the sampling rate sufficient?

      DT cutoff frequency $\Omega = \omega T_S = 2\pi(160)1.5\bullet10^{-3} \Rightarrow$ **$\Omega = 1.5079645$**

      Normalized DT cutoff frequency $\mathrm{Wn} = \Omega/\pi = 1.5079645/\pi \Rightarrow$ **Wn = 0.48**

      Since 0 < Wn = 0.48 < 1, the **sampling rate is sufficient**.

   b) Determine and list (each on separate line) the coefficient vectors a and b for the DT filter.

      From the MATLAB Command Window (extra line breaks removed, kept 6 digits)

      b = 0.011197 0.067184 0.167960 0.223947 0.167960 0.067184 0.011197

      a = 1.000000 -1.399302 2.120709 -1.791078 1.192879 -0.510949 0.125296

   c) Type out the corresponding DT I/O difference equation in standard form <snip> **and** in the recursive form <snip> What is the order N of the I/O difference equation? What is M?

      Standard form

$$y[n] - 1.3993\,y[n-1] + 2.1207\,y[n-2] - 1.7911\,y[n-3] + 1.1929\,y[n-4] - 0.5109\,y[n-5] + 0.1253\,y[n-6] =$$
$$0.0112\,x[n] + 0.0672\,x[n-1] + 0.168\,x[n-2] + 0.2239\,x[n-3] + 0.168\,x[n-4] + 0.0672\,x[n-5] + 0.0112\,x[n-6]$$

      Recursive form

$$y[n] = 1.39903\,y[n-1] - 2.1207\,y[n-2] + 1.7911\,y[n-3] - 1.1929\,y[n-4] + 0.5109\,y[n-5] - 0.1253\,y[n-6] +$$
$$0.0112\,x[n] + 0.0672\,x[n-1] + 0.168\,x[n-2] + 0.2239\,x[n-3] + 0.168\,x[n-4] + 0.0672\,x[n-5] + 0.0112\,x[n-6]$$

      From the $y[n-6]$ term, we know the order $N = 6$.

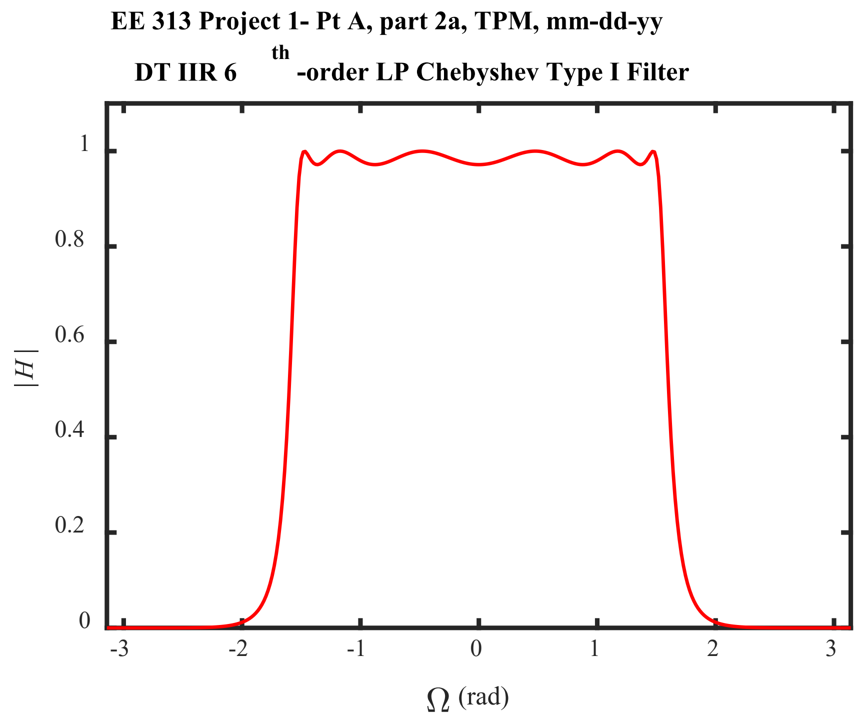
      From the $x[n-6]$ term, we can see that $M = 6$ as well.

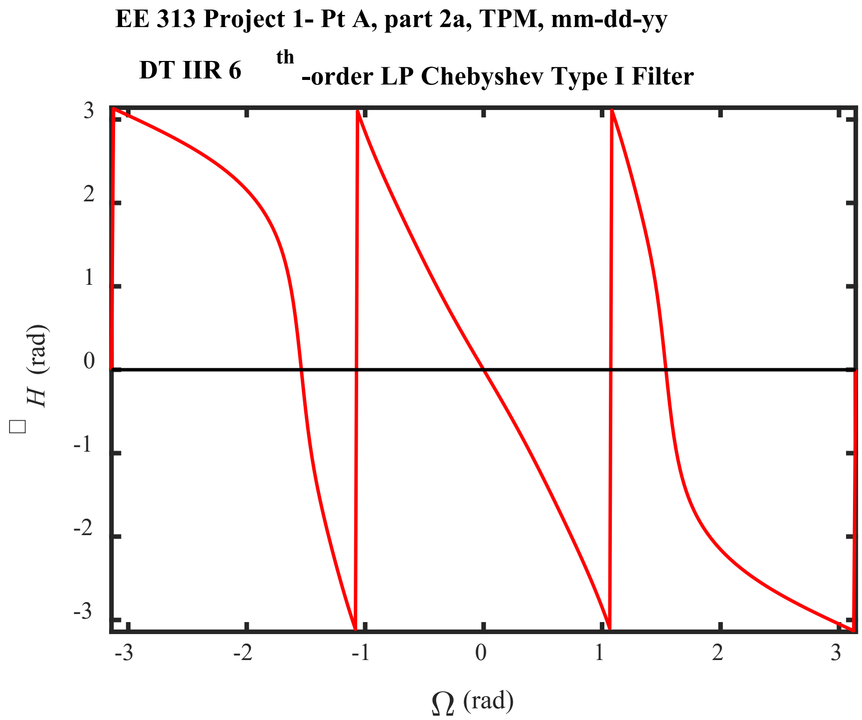
   d) Give a listing of the m-file.

      <snip>

2) Next, you will study the frequency response of the filter to ensure it meets the specifications. <snip> Use MATLAB (write an m-file) to perform the following tasks:

   a) Plot <snip> $|H|$ and phase $\angle H$ response <snip> vs Ω for $-\pi \le \Omega \le \pi$.<snip>





b) For the given sampling period, find the maximum CT frequencies ω_{max} (rad/s) and f_{max} (Hz) covered by this range of Ω.

Note: $\omega_{max} = \Omega_{max} / T_s = \pi / 1.5\text{e-}3$ and $f_{max} = \omega_{max} / (2\pi)$

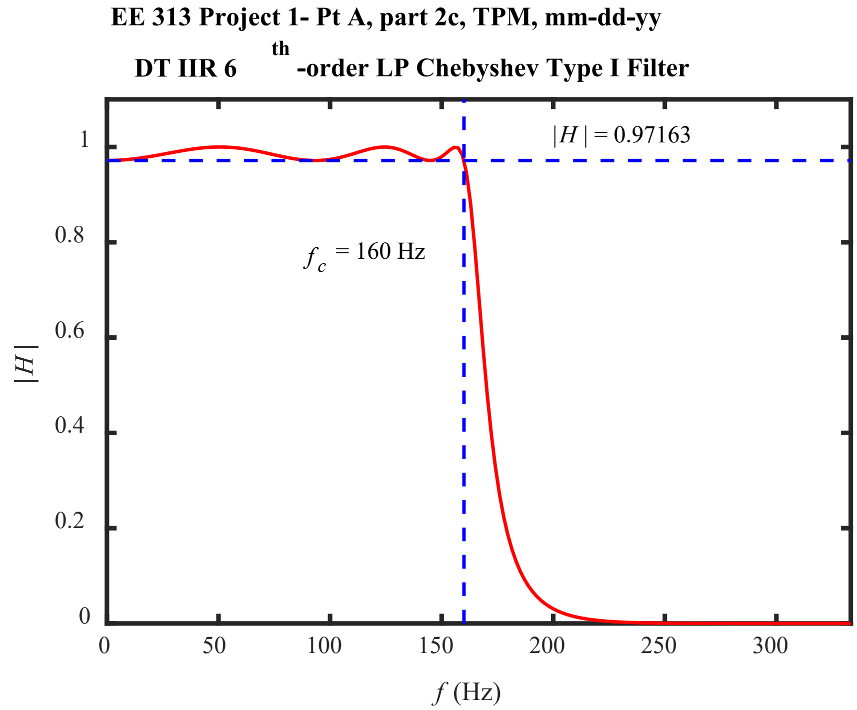
From the MATLAB Command Window (extra line breaks removed)

wmax = 2.094395102393195e+03

fmax = 3.333333333333333e+02

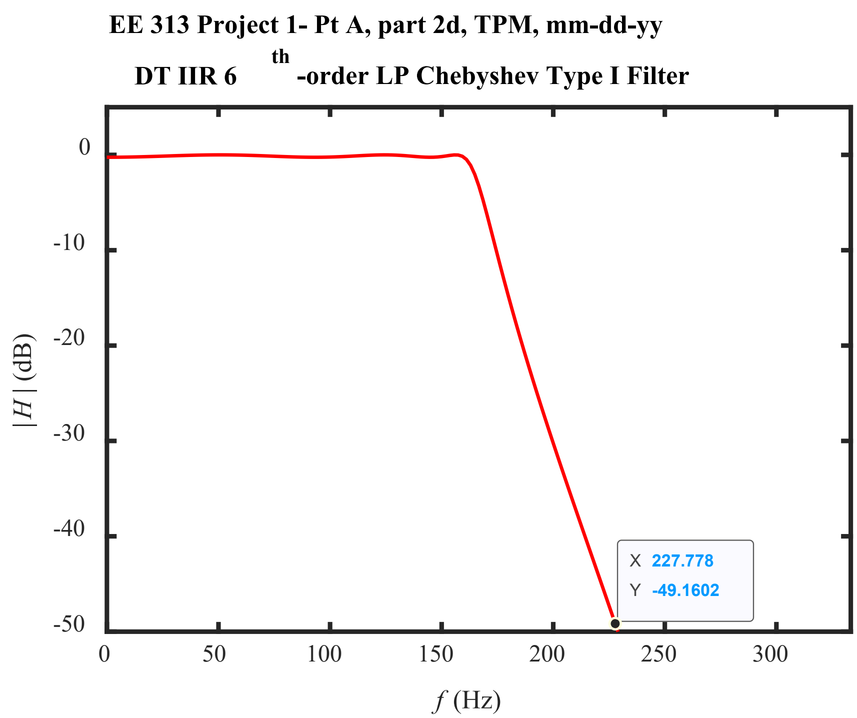
So, **ω_{max} = 2094.395 rad/s** and **f_{max} = 333.33 Hz**

c) Plot of the magnitude response of the filter $|H|$ versus f for $0 \le f \le f_{max}$. Use a vertical scale of 0 to 1.1. On the plot, place and label a horizontal dashed line at $|H| = 10^{-R/20}$ and a vertical dashed line at f_c. Do these lines intersect on the curve for $|H|$ as expected?



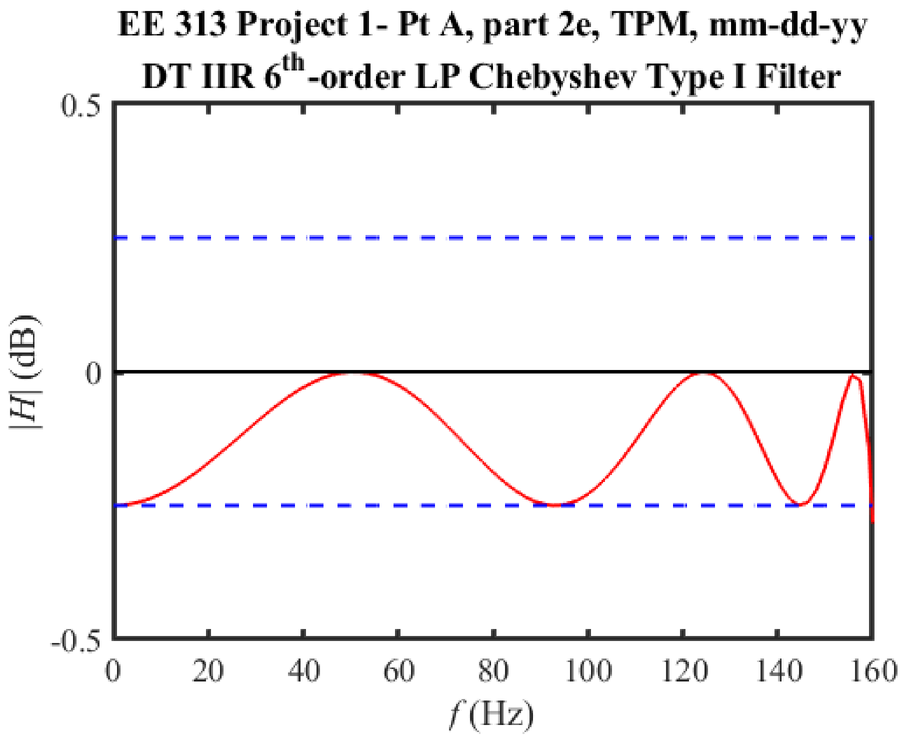
- Yes, the dashed lines intersect on |H| at f_c ! LP filter is working correctly.

d) Plot magnitude response of the filter $|H|$ **in decibels** vs f (Hz) for $0 \le f \le f_{max}$. Use a vertical scale of -50 to 5 dB. At what frequency f_{50} is $|H|$ down 50 dB? Express f_{50} in Hz & as a multiple of f_c.



- By interpolation, $|H|$ is down 50 dB at $f_{50} \approx$ **<u>229 Hz</u> = <u>1.43125 f_c</u>.**

e) Plot the magnitude response of the filter $|H|$ **in decibels** versus f (Hz) for $0 \le f \le f_c$. Use a vertical scale of -0.5 to 0.5 dB. On the plot, place labeled horizontal dashed lines at $\pm R$ (dB).



- Note the ripple $R = 0.25$ dB is only down from 0 dB!

f) Determine & tabulate the magnitude response (unitless and in dB) of the filter $|H(f)|$ when $f = f_c/3$, f_c, $1.1\,f_c$, and $1.25\,f_c$. Table format: col. 1 f (Hz), col. 2 $|H(f)|$, and col. 3 $|H(f)|$(dB).

From the MATLAB Command Window (extra line breaks removed)

Hdmag = 0.99975612218873 0.971627951577114 0.26904114404264 0.030779282453405

HdmagdB = -0.0021 -0.2500 -11.4036 -30.2348 at $f = f_c/3$, f_c, $1.1\,f_c$, and $1.25\,f_c$ respectively.

f **(Hz)**	**|*H(f)*|**	**|*H(f)*| (dB)**
$f_c/3$ = **53.33**	**0.99975612218873**	**-0.0021**
f_c = **160**	**0.971627951577114**	**-0.25**
$1.1\,f_c$ = **176**	**0.26904114404264**	**-11.4036**
$1.25\,f_c$ = **200**	**0.030779282453405**	**-30.2348**

g) Does the filter meet the design specifications? Explain why or why not.

- **Yes**, it has the correct ripple, cut-off frequency, and is of the appropriate order.

h) Give a listing of the m-file.

<snip>