EE 313 Signals and Systems (Fall 2024)
Project 1 Unit Pulse Response/Convolution Representation, Part A

Introduction

In this lab, we will examine finding the output of a discrete-time (DT) filter for a given input signal by convolution, i.e., convolve the input signal with the unit impulse response of the filter. For this lab, we will design a Chebyshev Type 1 DT lowpass (LP) filter. This type of filter has an infinite impulse response (IIR). However, to implement the convolution representation, we will truncate the unit pulse response $h[n]$ to be of length $N_{\max}$, i.e., $0 \le n \le N_{\max}$ -1. In essence, we will create a finite impulse response (FIR) DT HP filter based on an IIR DT HP filter.

Chebyshev Type I filters are characterized by a magnitude response that is equiripple in the passband and monotonic in the stopband (i.e., magnitude always decreases from low to high frequencies for the lowpass filter). Chebyshev Type I filters trade allowing a ripple in the passband for a faster roll off in the magnitude response. For more information on Chebyshev filters, see section 8.6.2 (analog) and section 10.3 (digital or DT) of the text.

Often DT filters are designed by first designing the appropriate analog filter (e.g., Butterworth, Chebyshev, …) and then mapping it to the DT domain by means of a bilinear transformation (see Chapter 10 of text). To design an N^{th}-order Chebyshev Type I DT IIR lowpass filter, use the MATLAB command 'cheby1', e.g., [b, a] = cheby1(N, R, Wp). Here, N is the order of the filter, R is the allowed or specified peak-to-peak ripple in the passband of the filter in decibels, and Wp is the <u>normalized</u> DT passband (cutoff) frequency. For Chebyshev Type I lowpass filters, the magnitude response of the transfer function H is down R (dB) at the cutoff frequency, i.e., $R\,(\text{dB}) = 20\log_{10}|H(f_c)|$ or $|H(f_c)| = 10^{-R/20}$ (unitless). To convert a continuous-time (CT) radian frequency $\omega = 2\pi f$ to the corresponding DT frequency Ω, use the relationship $\Omega = \omega T_S$ where T_S is the sampling period/rate. To normalize, divide Ω by π, i.e., Wp = Ω / π. If the signal is properly sampled, the value of the normalized DT cutoff frequency is 0 < Wp < 1. The output of the 'cheby1' command is two vectors a = [a_0 a_1 … a_n] and b = [b_0 b_1 … b_n] containing the coefficients of the DT I/O difference equation for the filter. Note that the coefficient a_0 is always 1. For more details on using this command, see the MATLAB tutorial by Kamen & Heck or use MATLAB help.

Project

1) Using a sampling period T_S = 1.5 ms, write an m-file to design a 6^{th}-order Chebyshev Type I DT IIR LP filter based on an analog cutoff frequency f_c = 160 Hz and passband ripple of R = 0.25 dB. The specific results desired are:
   a) Calculate and list the corresponding DT (Ω) and normalized (Wp) DT cutoff frequencies. Is the sampling rate sufficient?
   b) Determine and list (each on separate line) the coefficient vectors a and b for the DT filter.
   c) Type out the corresponding DT I/O difference equation in standard form, i.e., per (2.25) of the text, **and** in the recursive form, i.e., per (2.26) of the text, with all terms and coefficients included. What is the order N of the I/O difference equation? What is M?
   d) Give a listing of the m-file.

2) Next, you will study the frequency response of the filter to ensure it meets the specifications. To do so, you will need to compute the z-transform transfer function of the filter, $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{B(z)}{A(z)} = \dfrac{b_0 + b_1 z^{-1} + \dots b_M z^{-M}}{a_0 + a_1 z^{-1} + \dots a_N z^{-N}}$, at specified DT frequencies (covered in Chapter 7 of the text). When $z = e^{j\Omega}$, $H(z)$ is periodic with 2π interval. Typically, the magnitude $|H(z)|$ and phase $\angle H(z)$ response of DT filters are plotted over the DT frequency interval $-\pi \le \Omega \le \pi$. Further, the magnitude response is an even function about $\Omega = 0$. The Signal Processing Toolbox of MATLAB provides the function 'freqz' to perform this task, e.g., H = freqz(b, a, omega), where b and a are the coefficient vectors of the filter, omega is a vector containing the DT frequencies for which to calculate the frequency response, and H is a vector containing the frequency response at the specified DT frequencies. Note that the frequency response will be composed of complex numbers, i.e., $H = \mathrm{Re}(H) + j\,\mathrm{Im}(H) = |H| \angle H$. Use MATLAB (write an m-file) to perform the following tasks:
   a) Plot of the magnitude $|H|$ and phase $\angle H$ response of the filter versus Ω for $-\pi \le \Omega \le \pi$. Use vertical scales of 0 to 1.1 for $|H|$ and $-\pi$ to π for $\angle H$.
   b) For the given sampling period, find the maximum CT frequencies ω_{max} (rad/s) and f_{max} (Hz) covered by this range of Ω.
   c) Plot of the magnitude response of the filter $|H|$ versus f for $0 \le f \le f_{max}$. Use a vertical scale of 0 to 1.1. On the plot, place and label a horizontal dashed line at $|H| = 10^{-R/20}$ and a vertical dashed line at f_c. Do these lines intersect on the curve for $|H|$ as expected?
   d) Plot the magnitude response of the filter $|H|$ **in decibels** versus f (Hz) for $0 \le f \le f_{max}$. Use a vertical scale of -50 to 5 dB. At what frequency f_{50} is $|H|$ down 50 dB? Express f_{50} in Hertz and as a multiple of f_c.
   e) Plot the magnitude response of the filter $|H|$ **in decibels** versus f (Hz) for $0 \le f \le f_c$. Use a vertical scale of -0.5 to 0.5 dB. On the plot, place labeled horizontal dashed lines at $\pm R$ (dB).
   f) Determine and tabulate the magnitude response (unitless and in dB) of the filter $|H(f)|$ when $f = f_c/3, f_c, 1.1 f_c$, and $1.25 f_c$. Table format: col. 1 f (Hz), col. 2 $|H(f)|$, and col. 3 $|H(f)|$(dB).
   g) Does the filter meet the design specifications? Explain why or why not.
   h) Give a listing of the m-file.

Project report format

The results should be organized into a word-processed short report.

- In addition to syllabus HW format requirements, use font size ≥ 12 points and line spacing ≥ 1.1.
- Include: 1) cover page, 2) Introduction, 3) body (broken down into subsections based on the steps in project), and 4) Summary & Conclusions.
- Put the calculations, results, m-files, and plots/figures **in the body** of the report in the order specified as they occur. Do not use appendices.
- On all plots, label horizontal and vertical axes, and insert a horizontal axis at 0. Put "EE 313, Project # & part #, *your initials*, date" in the title.
- For all m-files, put the filename, EE 313, Project # & part #, *your name*, and date in comment lines.
- Staple results together and turn-in the project report on **Friday, October 4, 2024**.