EE 313 Signals and Systems (Fall 2024) Examination 1

Name __KEY B__

Instructions: Show all work for full credit. Write answers in indicated places. Insert equation sheet inside exam.

1) A discrete-time signal $x[n] = -2\,p_3[n-1]$ is fed into a system with a unit-pulse response given by $h[n] = \delta[n-1] + 0.8\,\delta[n-2] + 0.4\,\delta[n-3] - 0.4\,\delta[n-4]$. Compute the output $y[n]$ from the system. Then, in the indicated places, sketch labeled stem plots of $x[n]$, $h[n]$, and $y[n]$ for $-1 \le n \le 8$.

	-2	-2	-2
1	-2	-2	-2
0.8	-1.6	-1.6	-1.6
0.4	-0.8	-0.8	-0.8
-0.4	0.8	0.8	0.8

$P + Q = 1 + 0 = 1$

$y[1] = -2$

$y[2] = -2 - 1.6 = -3.6$

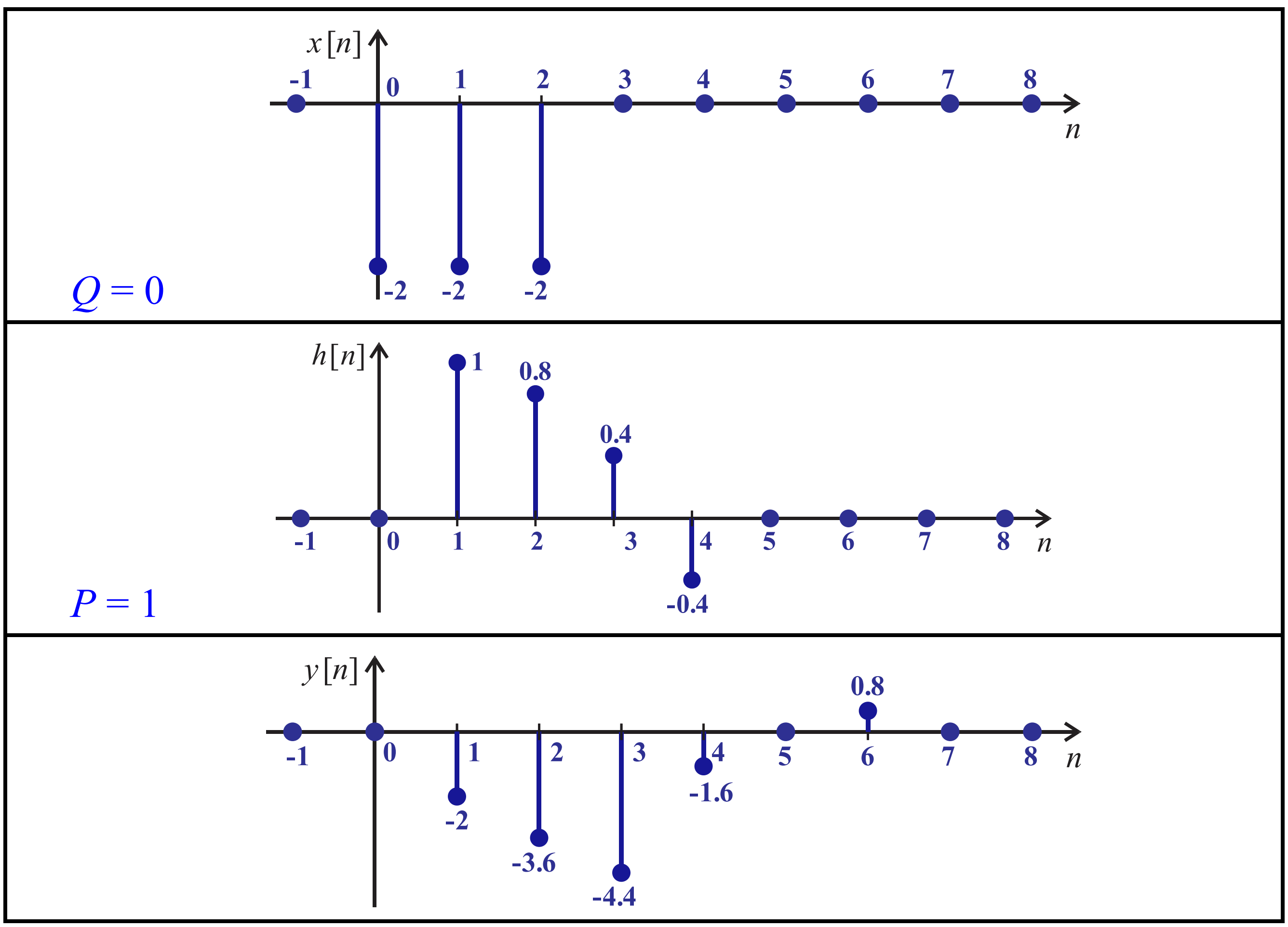
$y[3] = -2 - 1.6 - 0.8 = -4.4$

$y[4] = -1.6 - 0.8 + 0.8 = -1.6$

$y[5] = -0.8 + 0.8 = 0$

$y[6] = 0.8$

$y[n] = 0$ for all other n



2) Answer/solve the following questions/problems.

a) Is the discrete-time function $w[n] = 15\cos(0.25\pi n + 0.2)$ **periodic** or **non-periodic**? (circle correct answer) **Show** why or why not.

From $w[n]$ equation, the DT frequency $\Omega = 0.25\pi$. Per section 1.2.4, if $\Omega/2\pi = 0.25\pi/2\pi = 0.125 = q/r$, an integer ratio, the function $w[n]$ is periodic. Since $1/8 = 0.125$, the ratio can be put in terms of the ratio of integers $\Rightarrow$ ***w[n] is periodic***.

b) Given the system output $y(t) = 2t^3 x(t) - 2x(t-4.2)$ for an input $x(t)$, is the system **additive** or **non-additive**? (circle correct answer) **Show** why or why not.

For input $x_1(t)$, $y_1(t) = 2t^3 x_1(t) - 2x_1(t-4.2)$.

For input $x_2(t)$, $y_2(t) = 2t^3 x_2(t) - 2x_2(t-4.2)$.

For input $x_1(t) + x_1(t)$, $\tilde{y}(t) = 2t^3\left[x_1(t) + x_2(t)\right] - 2\left[x_1(t-4.2) + x_2(t-4.2)\right] = y_1(t) + y_2(t)$

$\Rightarrow$ **system is additive**.

c) Given the system output $y(t) = 2t^3 x(t) - 2x(t-4.2)$ for an input $x(t)$, is the system **homogenous** or **inhomogeneous**? (circle correct answer) **Show** why or why not.

Note that $ay(t) = a\left[2t^3 x(t) - 2x(t-4.2)\right]$.

For input $ax(t)$, $\tilde{y}(t) = 2t^3\left[ax(t)\right] - 2\left[ax(t-4.2)\right] = a\left[2t^3 x(t) - 2x(t-4.2)\right] = ay(t)$

$\Rightarrow$ **system is homogeneous**.

d) Given the system output $y(t) = 2t^3 x(t) - 2x(t-4.2)$ for an input $x(t)$, is the system **linear** or **nonlinear**? (circle correct answer) **Show** why or why not.

Since the system is **additive** and **homogenous** $\Rightarrow$ **system is linear**.

e) Is the system described by $v(t) = -5t^3 x(t+1) + 5x(t-2.2)$ **causal** or **noncausal**? Does the system have **memory** or is it **memoryless**? (circle correct answers)

Why or why not?

System is **noncausal** because of the $x(t+1)$ term depending on a future value of the input $x(t)$.

System has **memory** as evidenced by the $x(t-2.2)$ term depending on a past value of the input $x(t)$.

3) A linear, time-invariant series *RLC* circuit has response $y(t) = 6 + 2e^{-4t} - 8e^{-2t}$ (V) for $t \geq 0$ to the input $x(t) = 0.5u(t)$ (V) when initially at rest. Determine the unit step response $g(t)$ and the impulse response $h(t)$ of this circuit. With the system initially at rest, determine the system output $y_1(t)$ to the input $x_1(t) = 8u(t-2)$ (V) and find $y_1(t = 3s)$.

Note that $2x(t) = u(t)$.

$\Rightarrow$ By linearity, $g(t) = 2y(t) = 2\left[6 + 2e^{-4t} - 8e^{-2t}\right] = 12 + 4e^{-4t} - 16e^{-2t}$ (V) for $t \geq 0$, or
$g(t) = \left[\ 12 + 4e^{-4t} - 16e^{-2t}\right]u(t)$ (V).

From notes, $h(t) = \dfrac{d\,g(t)}{dt} = 0 + 4(-4)e^{-4t} - 16(-2)e^{-2t} = -16e^{-4t} + 32e^{-2t}$ (V/s) for $t > 0$,
or $h(t) = \left[-16e^{-4t} + 32e^{-2t}\right]u(t)$ (V/s).

Using the convolution representation,

$$y_1(t) = x_1(t) * h(t) = 8u(t-2) * \left(-16e^{-4t} + 32e^{-2t}\right)u(t).$$

However, that would be the **hard way** to find the answer.

An **easier** solution method is to note that input $x_1(t) = 8u(t-2)$ is a weighted and time-shifted unit step function. Using **linearity and time-invariance on the unit step response $g(t)$**,

$$\begin{aligned} y_1(t) &= 8g(t-2) = 8\left[12 + 4e^{-4t} - 16e^{-2t}\right]u(t)\Big|_{t\to t-2} \\ &= \left[96 + 32e^{-4(t-2)} - 128e^{-2(t-2)}\right]u(t-2) \text{ (V)} \quad . \\ &= \left[96 + 32e^{8}e^{-4t} - 128e^{4}e^{-2t}\right]u(t-3) \text{ (V)} \end{aligned}$$

Evaluating $y_1(t)$ at $t = 3$ s, yields

$$\begin{aligned} y_1(t = 3\text{ s}) &= \left[96 + 32e^{-4(3-2)} - 128e^{-2(3-2)}\right]u(3-2) \text{ (V)} \\ &= \left[96 + 32e^{-4} - 128e^{-2}\right](1) \\ &= 79.26318 \text{ (V)} \end{aligned}$$

$g(t) = \underline{\left[\ 12 + 4e^{-4t} - 16e^{-2t}\right]u(t)\ \text{(V)}}$ $h(t) = \underline{\left[-16e^{-4t} + 32e^{-2t}\right]u(t)\ \text{(V/s)}}$

$y_1(t) = \underline{\left[96 + 32e^{-4(t-2)} - 128e^{-2(t-2)}\right]u(t-2)\ \text{(V)}}$ $y_1(t = 4s) =$ __79.26318 (V)__

4) Using backward difference approximations, discretize the differential equation $2.4\frac{d^2y(t)}{dt^2}+6\frac{dy(t)}{dt}-3y(t)=8x(t)$ for $t \ge 0$ given a sampling rate $T=0.4$ s and input $x(t)=2u(t)$. Put the resulting difference equation in recursive form, i.e., $y[n]=-\sum_{i=1}^{N} a_i y[n-i]+\sum_{i=0}^{M} b_i x[n-i]$, and list the range of the index n. Given $y(0)=-6$ and $\left.\frac{dy(t)}{dt}\right|_{t=0}=-8$, determine the discrete-time initial conditions. Calculate the system output $y[n]$ at $n=2$.

Discretizing the I/O differential equation for an input $x(t)=2u(t)$, yields

$$2.4\left[\frac{y[n]-2y[n-1]+y[n-2]}{T^2}\right]+6\left[\frac{y[n]-y[n-1]}{T}\right]-3y[n]=8x[n]=8\big(2u[n]\big)=16u[n].$$

Substituting in $T = 0.4$ s and doing some algebra, yields

$$(15+15-3)y[n]+(-30-15)y[n-1]+15y[n-2]=16u[n].$$

Then, putting this I/O difference equation into recursive form yields

$$y[n]=\frac{45}{27}y[n-1]-\frac{15}{27}y[n-2]+\frac{16}{27}u[n]=1.\overline{66}\,y[n-1]-0.\overline{55}\,y[n-2]+0.5925926u[n].$$

Next, we need to find the discrete-time initial conditions.

Discretizing $y(0) = -6$ $\Rightarrow$ $\underline{y[0] = -6}$ $\Rightarrow$ $y[n]$ recursion is good for $\boldsymbol{n \ge 1}$.

Discretizing $\left.\frac{dy(t)}{dt}\right|_{t=0}=-8$ yields-

$$\left.\frac{y[0]-y[-1]}{T}\right|_{T=0.4}=\frac{-6-y[-1]}{0.4}=-8 \qquad \Rightarrow \underline{y[-1] = -2.8}.$$

Using recursion,

For $n = 1$, $y[n=1]=y[1]=\frac{45}{27}y[1-1]-\frac{15}{27}y[1-2]+\frac{16}{27}u[1]=\frac{45}{27}(-6)-\frac{15}{27}(-2.8)+\frac{16}{27}=-7.85\overline{185}$.

For $n = 2$,

$$y[n=2]=y[2]=\frac{45}{27}y[2-1]-\frac{15}{27}y[2-2]+\frac{16}{27}u[2]=\frac{45}{27}(-7.85\overline{185})-\frac{15}{27}(-6)+\frac{16}{27}=-9.160494.$$

$y[n]$ = $\underline{1.\overline{66}\,y[n-1]-0.\overline{55}\,y[n-2]+0.592593u[n]}$ **for $\boldsymbol{n \ge 1}$**

Discrete-time initial conditions: _<u>y[0] = -6 and y[-1] = -2.8</u> $y[2]$ = <u>-9.1605</u>