EE 313 Signals and Systems (Fall 2024) Examination 1

Name _KEY A_

Instructions: Show all work for full credit. Write answers in indicated places. Insert equation sheet inside exam.

1) A discrete-time signal $x[n] = -4p_3[n-2]$ is fed into a system with a unit-pulse response given by $h[n] = \delta[n] + 0.6\,\delta[n-1] + 0.2\,\delta[n-2] - 0.3\,\delta[n-3]$. Compute the output $y[n]$ from the system. Then, in the indicated places, sketch labeled stem plots of $x[n]$, $h[n]$, and $y[n]$ for $-1 \le n \le 8$.

	-4	-4	-4
1	-4	-4	-4
0.6	-2.4	-2.4	-2.4
0.2	-0.8	-0.8	-0.8
-0.3	1.2	1.2	1.2

$P + Q = 0 + 1 = 1$

$y[1] = -4$

$y[2] = -4 - 2.4 = -6.4$

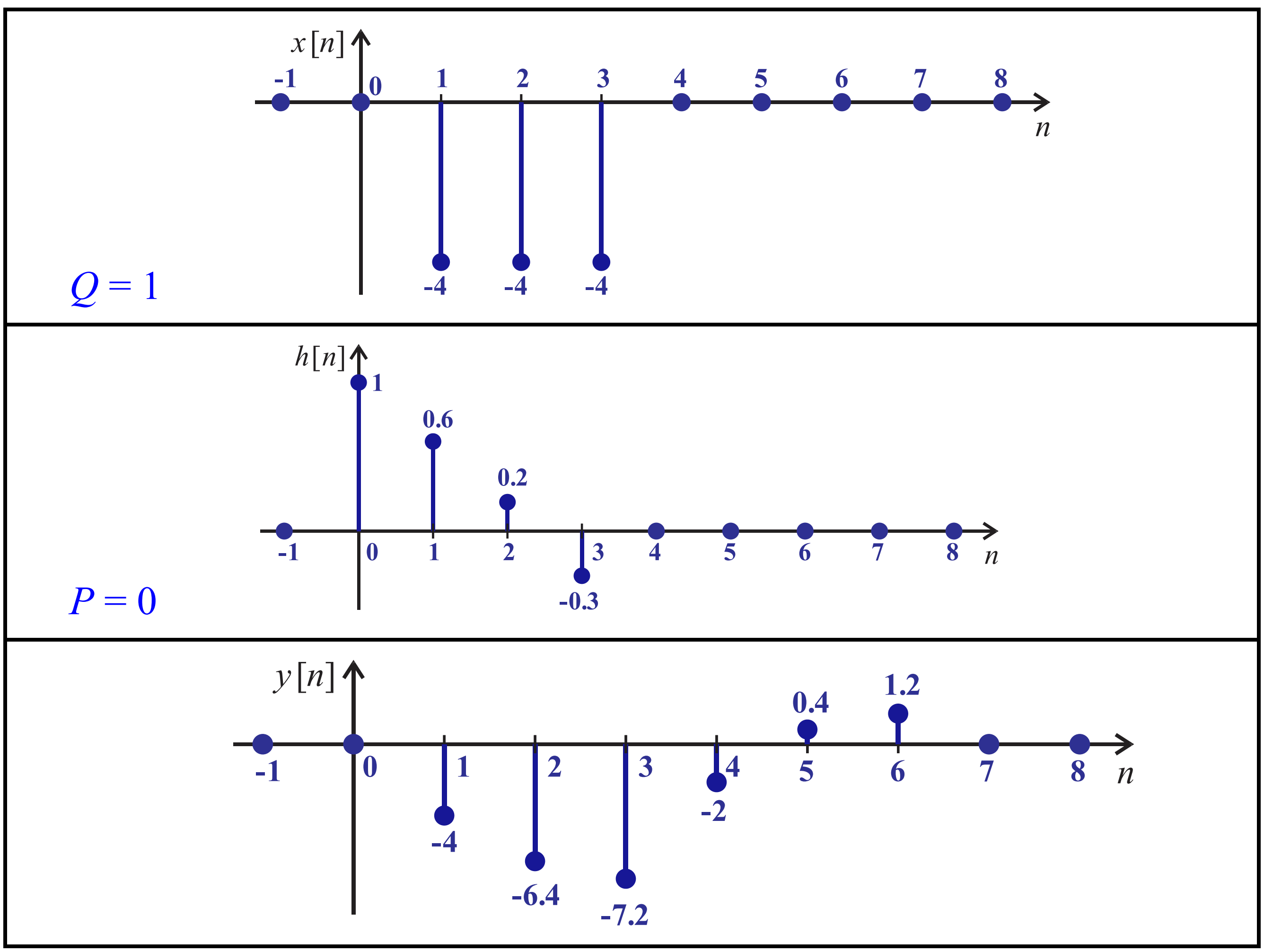
$y[3] = -4 - 2.4 - 0.8 = -7.2$

$y[4] = -2.4 - 0.8 + 1.2 = -2$

$y[5] = -0.8 + 1.2 = 0.4$

$y[6] = 1.2$

$y[n] = 0$ for all other n



2) Answer/solve the following questions/problems.

a) Is the discrete-time function $w[n] = 25\cos(5n - 0.2\pi)$ **periodic** or **non-periodic**? (circle correct answer) **Show** why or why not.

From $w[n]$ equation, the DT frequency $\Omega = 5$. Per section 1.2.4, if $\Omega/2\pi = 5/2\pi = q/r$, an integer ratio, the function $w[n]$ is periodic. Since π is irrational, the ratio cannot be put in terms of the ratio of integers $\Rightarrow$ **$w[n]$ is non-periodic**.

b) Given the system output $y(t) = -4t^2x(t) + 2x(t-2.2)$ for an input $x(t)$, is the system **additive** or **non-additive**? (circle correct answer) **Show** why or why not.

For input $x_1(t)$, $y_1(t) = -4t^2x_1(t) + 2x_1(t-2.2)$.

For input $x_2(t)$, $y_2(t) = -4t^2x_2(t) + 2x_2(t-2.2)$.

For input $x_1(t) + x_1(t)$, $\tilde{y}(t) = -4t^2\left[x_1(t) + x_2(t)\right] + 2\left[x_1(t-2.2) + x_2(t-2.2)\right] = y_1(t) + y_2(t)$

$\Rightarrow$ **system is additive**.

c) Given the system output $y(t) = -4t^2x(t) + 2x(t-2.2)$ for an input $x(t)$, is the system **homogenous** or **inhomogeneous**? (circle correct answer) **Show** why or why not.

Note that $ay(t) = a\left[-4t^2x(t) + 2x(t-2.2)\right]$.

For input $ax(t)$, $\tilde{y}(t) = -4t^2\left[ax(t)\right] + 2\left[ax(t-2.2)\right] = a\left[-4t^2x(t) + 2x(t-2.2)\right] = ay(t)$

$\Rightarrow$ **system is homogeneous**.

d) Given the system output $y(t) = -4t^2x(t) + 2x(t-2.2)$ for an input $x(t)$, is the system **linear** or **nonlinear**? (circle correct answer) **Show** why or why not.

Since the system is **additive** and **homogenous** $\Rightarrow$ **system is linear**.

e) Is the system described by $w(t) = 8t^2x(t-1) - 4x(t+2.2)$ **causal** or **noncausal**? Does the system have **memory** or is it **memoryless**? (circle correct answers)

Why or why not?

System is **noncausal** because of the $x(t+2.2)$ term depending on a future value of the input $x(t)$.

System has **memory** as evidenced by the $x(t-1)$ term depending on a past value of the input $x(t)$.

3) A linear, time-invariant series *RLC* circuit has response $y(t) = 4.5 + 1.5e^{-4t} - 6e^{-2t}$ (V) for $t \geq 0$ to the input $x(t) = 0.25u(t)$ (V) when initially at rest. Determine the unit step response $g(t)$ and the impulse response $h(t)$ of this circuit. With the system initially at rest, determine the system output $y_1(t)$ to the input $x_1(t) = 6u(t-3)$ (V) and find $y_1(t = 4\,s)$

Note that $4x(t) = u(t)$.

$\Rightarrow$ By linearity, $g(t) = 4y(t) = 4\left[4.5 + 1.5e^{-4t} - 6e^{-2t}\right] = 18 + 6e^{-4t} - 24e^{-2t}$ (V) for $t \geq 0$,

or $g(t) = \left[18 + 6e^{-4t} - 24e^{-2t}\right]u(t)$ (V).

From notes, $h(t) = \dfrac{d\,g(t)}{dt} = 0 + 6(-4)e^{-4t} - 24(-2)e^{-2t} = -24e^{-4t} + 48e^{-2t}$ (V/s) for $t > 0$,

or $h(t) = \left[-24e^{-4t} + 48e^{-2t}\right]u(t)$ (V/s).

Using the convolution representation,

$$y_1(t) = x_1(t) * h(t) = 6u(t-3) * \left(-24e^{-4t} + 48e^{-2t}\right)u(t).$$

However, that would be the **hard way** to find the answer.

An **easier** solution method is to note that input $x_1(t) = 6u(t-3)$ is a weighted and time-shifted unit step function. Using **linearity and time-invariance on the unit step response *g*(*t*)**,

$$y_1(t) = 6g(t-3) = 6\left[18 + 6e^{-4t} - 24e^{-2t}\right]u(t)\Big|_{t \to t-3}$$

$$= \left[108 + 36e^{-4(t-3)} - 144e^{-2(t-3)}\right]u(t-3) \text{ (V)}.$$

$$= \left[108 + 36e^{12}e^{-4t} - 144e^{6}e^{-2t}\right]u(t-3) \text{ (V)}$$

Evaluating $y_1(t)$ at $t = 4$ s, yields

$$y_1(t = 4\text{ s}) = \left[108 + 36e^{-4(4-3)} - 144e^{-2(4-3)}\right]u(4-3) \text{ (V)}$$

$$= \left[108 + 36e^{-4} - 144e^{-2}\right](1)$$

$$= 89.17108 \text{ (V)}$$

$g(t)$ = $\left[18 + 6e^{-4t} - 24e^{-2t}\right]u(t)$ (V) $h(t)$ = $\left[-24e^{-4t} + 48e^{-2t}\right]u(t)$ (V/s)

$y_1(t)$ = $\left[108 + 36e^{-4(t-3)} - 144e^{-2(t-3)}\right]u(t-3)$ (V) $y_1(t = 4s)$ = **89.17108 (V)**

4) Using backward difference approximations, discretize the differential equation $1.8\frac{d^2y(t)}{dt^2}-6\frac{dy(t)}{dt}+3y(t)=9x(t)$ for $t\geq 0$ given a sampling rate $T=0.3$ s and input $x(t)=2u(t)$. Put the resulting difference equation in recursive form, i.e., $y[n]=-\sum_{i=1}^{N}a_i y[n-i]+\sum_{i=0}^{M}b_i x[n-i]$, and list the range of the index n. Given that $y(0)=-2$ and $\left.\frac{dy(t)}{dt}\right|_{t=0}=-4$, determine the discrete-time initial conditions. Calculate the system output $y[n]$ at $n=2$.

Discretizing the I/O differential equation for an input $x(t)=2u(t)$, yields

$$1.8\left[\frac{y[n]-2y[n-1]+y[n-2]}{T^2}\right]-6\left[\frac{y[n]-y[n-1]}{T}\right]+3y[n]=9x[n]=9\big(2u[n]\big)=18u[n].$$

Substituting in $T=0.3$ s and doing some algebra, yields

$$(20-20+3)y[n]+(-40+20)y[n-1]+20y[n-2]=18u[n].$$

Then, putting this I/O difference equation into recursive form yields

$$y[n]=\frac{20}{3}y[n-1]-\frac{20}{3}y[n-2]+6u[n]=6.\overline{66}\,y[n-1]-6.\overline{66}\,y[n-2]+6u[n] \text{ for } n\geq 1.$$

Next, we need to find the discrete-time initial conditions.

Discretizing $y(0)$ = -2 $\Rightarrow$ $y[0] = -2$ $\Rightarrow$ $y[n]$ recursion is good for $\boldsymbol{n\geq 1}$.

Discretizing $\left.\frac{dy(t)}{dt}\right|_{t=0}=-4$ yields

$$\left.\frac{y[0]-y[-1]}{T}\right|_{T=0.3}=\frac{-2-y[-1]}{0.3}=-4 \Rightarrow \underline{y[-1]=-0.8}.$$

Using recursion,

For $n=1$, $y[n=1]=y[1]=\frac{20}{3}y[1-1]-\frac{20}{3}y[1-2]+6u[1]=6.\overline{66}(-2)-6.\overline{66}(-0.8)+6=-2$.

For $n=2$, $y[n=2]=y[2]=\frac{20}{3}y[2-1]-\frac{20}{3}y[2-2]+6u[2]=6.\overline{66}(-2)-6.\overline{66}(-2)+6=6$.

$y[n]$ = $\underline{6.\overline{66}\,y[n-1]-6.\overline{66}\,y[n-2]+6u[n]}$ **for $n\geq 1$**

Discrete-time initial conditions: _$y[0]$ = -2 _and $y[-1]$ = -0.8 $y[2]$ = _6_