ex. Find the step response, $x[n]=u[n]$, of the filter $H(z)=\frac{z^2-0.9}{z^2+0.81}$

$$X(z)=\frac{z}{z-1} \rightarrow Y(z)=X(z)H(z)$$

$$Y(z)=\frac{z^3-0.9z}{(z-1)(z^2+0.81)}=\frac{z^3-0.9z}{z^3-z^2+0.81z-0.81}$$

could use to find I/O Diff. eqn

$$\frac{Y(z)}{z}=\frac{z(z^2-0.9)}{z(z^3-z^2+0.81z-0.81)}=\frac{z^2-0.9}{(z-j0.9)(z+j0.9)(z-1)}$$

$$=\frac{C_1}{z-j0.9}+\frac{C_2}{z+j0.9}+\frac{C_3}{z-1}$$

$$C_1=\left[(z-j0.9)\frac{z^2-0.9}{(z-j0.9)(z+j0.9)(z-1)}\right]_{z=j0.9=0.9\angle\frac{\pi}{2}=P_1}=0.70613\angle -48.0128^\circ$$

$$C_2=C_1^*=0.70613\angle 48.0128^\circ=0.4724+j0.52486$$

$$C_3=\left[(z-1)\frac{z^2-0.9}{(z-j0.9)(z+j0.9)(z-1)}\right]_{z=1}=\frac{1}{18.1}=0.055249$$

$$\frac{Y(z)}{z}=\frac{0.70613\angle -48.013^\circ}{z-j0.9}+\frac{0.70613\angle +48.013^\circ}{z+j0.9}+\frac{1/18.1}{z-1}$$

$$Y(z)=\frac{0.70613\angle -48.013^\circ z}{z-j0.9}+\frac{0.70613\angle 48.013^\circ z}{z+j0.9}+\frac{1/18.1\ z}{z-1}$$

$|C_1|$ $|P_1|$ $\angle P_1$ $-48.013^\circ \times \frac{\pi}{180^\circ}$

$$y[n] = 2(0.70613)\,0.9^n \cos\left(\frac{\pi}{2}n - 0.83798\right)u[n] + \frac{1}{18.1}u[n]$$

$$y[n\to\infty] = \underline{1/18.1 = 0.055249}$$

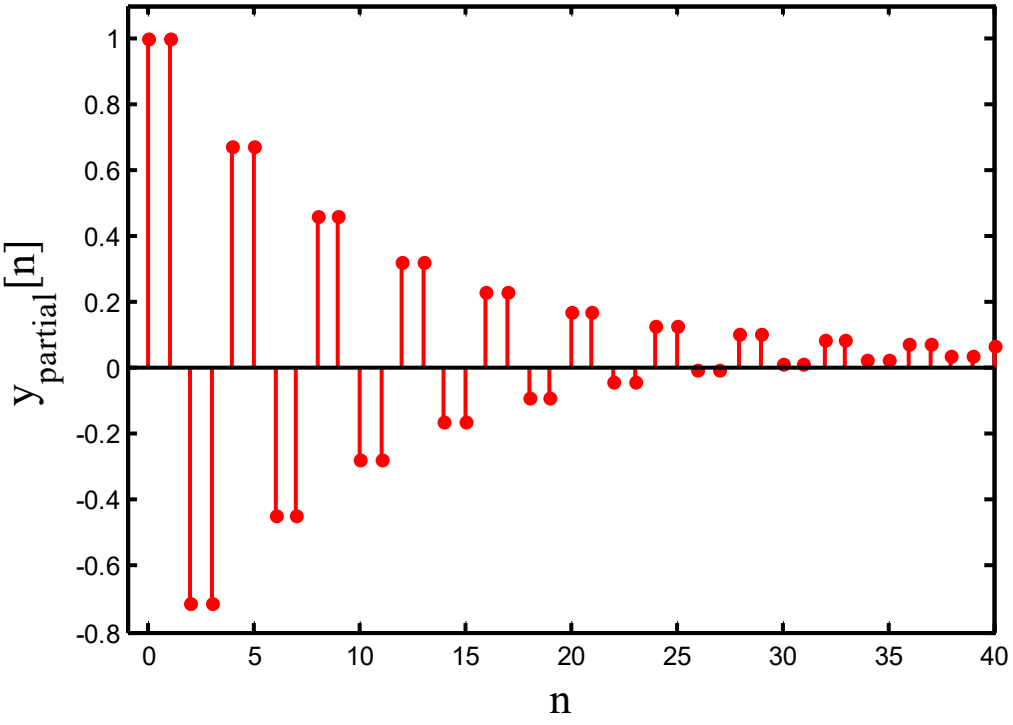
Step Response can also be computed numerically from $H(z)$

```
num = [1, 0, -0.9];   % Coeff. of numerator of H(z)  z^2 + 0z - 0.9
den = [1, 0, 0.81];   % Coeff. of denominator of H(z)  z^2 + 0z + 0.81
n = 0:50;
y = dstep(num, den, 41);   % finds step response
```

```
% chap7_z_transform_system_response.m
% Find system step Response using z-transform
% Compute system response where H(z)=[z^2-0.9]/[z^2+0.81]
% to a unit step x[n]=u[n] --> X(z)= z/(z-1).
% This yields Y(z) = X(z)H(z) = [z/(z-1)](z^2-0.9)/(z^2+0.81)
%
close all; clc; clear;
num = [1,0,-0.9];   % Coefficients of numerator of H(z)
den = [1,0,0.81];   % Coefficients of denominator of H(z)
n=0:1:40;
y_long = dstep(num,den,41);
y_part = 2*0.70613*(0.9.^n).*cos(pi*n/2-0.83798) + 1/18.1;
stem(n,y_part,'r.'),axis([-1 40 -0.8 1.1]),
ylabel('y_{partial }[n]','fontsize',16,'fontname','times'),
xlabel('n','fontsize',16,'fontname','times'),
title({'System Step Response using z-Transform- partial fractions';...
    'Y(z) = X(z) H(z) = [z/(z-1)]*(z^2-0.9)/(z^2+0.81)'},...
    'fontsize',15,'fontname','times'),
figure,stem(n,y_long,'k.'),axis([-1 40 -0.8 1.1]),
ylabel('y_{long }[n]','fontsize',16,'fontname','times'),
xlabel('n','fontsize',16,'fontname','times'),
title({'System Step Response using z-Transform- long division';...
    'Y(z) = X(z) H(z) = [z/(z-1)]*(z^2-0.9)/(z^2+0.81)'},...
    'fontsize',15,'fontname','times'),
set(findobj('type','line'),'linewidth',1.5)
set(findobj('type','line'),'markersize',16)
set(findobj('type','axes'),'linewidth',2)
set(findobj('type','text'),'fontsize',12,'fontname','times')
```

System Step Response using z-Transform- partial fractions

$Y(z) = X(z)\,H(z) = [z/(z-1)]*(z^2-0.9)/(z^2+0.81)$



System Step Response using z-Transform- long division

$Y(z) = X(z)\,H(z) = [z/(z-1)]*(z^2-0.9)/(z^2+0.81)$