Example- Find the system transfer function $H(z)$ using Mason's Theorem.
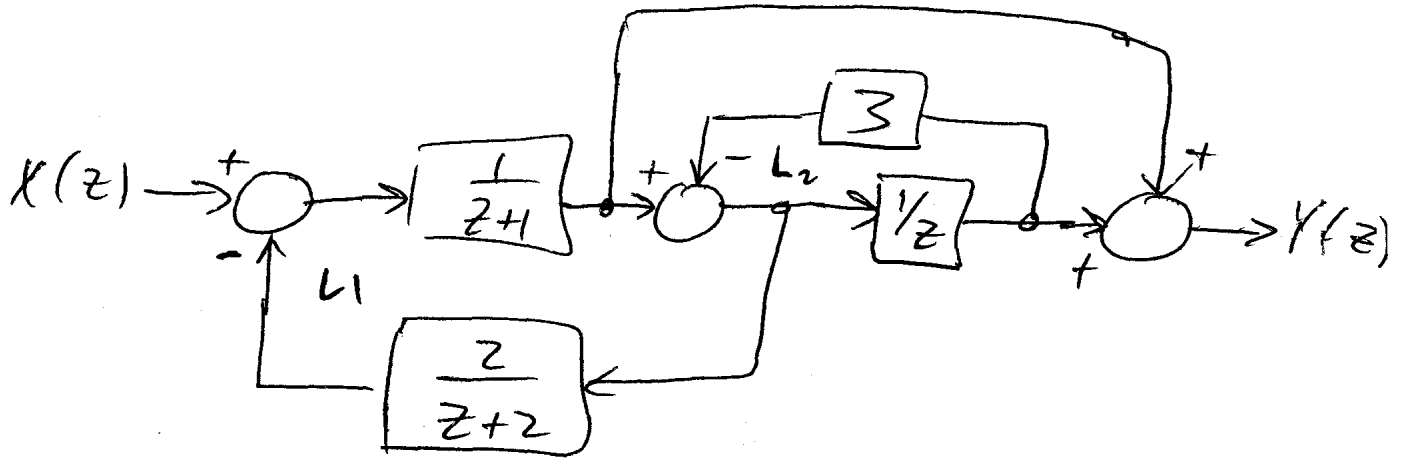

$$P_1(z) = \left(\frac{1}{z+1}\right) \qquad (Top)$$

$$P_2(z) = \left(\frac{1}{z+1}\right)\left(\frac{1}{z}\right) \qquad (Middle)$$

$$L_1(z) = \left(\frac{1}{z+1}\right)\left(\frac{-2}{z+2}\right) \qquad Left$$

$$L_2(z) = \left(\frac{1}{2}\right)(-3) = -\frac{3}{z} \qquad Right$$

$\Big\}$ Touch

$$H(z) = \frac{\sum\limits_{i=1}^{2} P_i(z)\,\Delta_i(z)}{\Delta(z)}$$

where $\Delta(z) = 1 - \left(L_1(z) + L_2(z)\right)$

$$= 1 + \frac{2}{(z+1)(z+2)} + \frac{3}{z}$$

$P_1(z)$ touches $L_1(z)$ but __Not__ $L_2(z)$

$$\Rightarrow \Delta_1(z) = 1 - L_2(z) = 1 + \frac{3}{z}$$

$P_2(z)$ touches both $L_1(z)$ & $L_2(z)$

$$\Rightarrow \Delta_2(z) = 1$$

$$H(z) = \frac{\left(\frac{1}{z+1}\right)\left(1 + \frac{3}{z}\right) + \left[\left(\frac{1}{z+1}\right)\left(\frac{1}{z}\right)\right](1)}{1 + \frac{z}{(z+1)(z+2)} + \frac{3}{z}}$$

$$= \frac{(z+2)(z+3) + (z+2)}{z(z+1)(z+2) + zz + 3(z+1)(z+2)}$$

$$= \frac{z^2 + 5z + 6 + z + 2}{z(z^2+3z+2) + 2z + 3(z^2+3z+2)}$$

$$= \frac{z^2 + 6z + 8}{z^3 + 3z^2 + 2z + 2z + 3z^2 + 9z + 6}$$

$$H(z) = \frac{z^2 + 6z + 8}{z^3 + 6z^2 + 13z + 6}$$