ex. $H(z) = \dfrac{z^2+6z+8}{z^3+7z^2+15z+9} = \dfrac{z^2+6z+8}{(z+3)^2(z+1)}$

poles $p_1 = -3$, repeated twice $r=2$

$p_2 = -1$

$$\frac{H(z)}{z} = \frac{z^2+6z+8}{z(z+1)(z+3)^2} = \frac{C_0}{z} + \frac{C_1}{z+3} + \frac{C_2}{(z+3)^2} + \frac{C_3}{z+1}$$

$$C_0 = H(0) = \frac{0+0+8}{0+0+0+9} = \underline{\underline{\frac{8}{9} = 0.\overline{88}}}$$

$$C_3 = \left[(z+1)\,\frac{z^2+6z+8}{z(z+1)(z+3)^2}\right]\Bigg|_{z=-1} = \frac{(-1)^2+6(-1)+8}{(-1)(-1+3)^2} = \underline{\underline{-0.75}}$$

$$C_2 = \left[(z+3)^2\,\frac{z^2+6z+8}{z(z+1)(z+3)^2}\right]_{z=-3} = \frac{(-3)^2+6(-3)+8}{-3(-3+1)} = \underline{\underline{-\frac{1}{6}}}$$

$$= \underline{\underline{-0.1\overline{66}}}$$

$$C_1 = \frac{1}{1!}\left[\frac{d}{dz}\left\{(z+3)^2\,\frac{z^2+6z+8}{z(z+1)(z+3)^2}\right\}\right]\Bigg|_{z=-3}$$

$$= \left[\frac{z(z+1)(2z+6) - (z^2+6z+8)(2z+1)}{z^2(z+1)^2}\right]\Bigg|_{z=-3}$$

$$= \frac{-3(-3+1)\overbrace{(2(-3)+6)}^{0} - (-3^2+6(-3)+8)(2(-3)+1)}{-3^2(-3+1)^2} = \underline{\underline{\frac{-5}{36} = -0.1\overline{38}}}$$

check w/ MATLAB

$$\frac{H(z)}{z} = \frac{z^2+6z+8}{z^4+7z^3+15z^2+9z+0}$$

```
num = [1, 6, 8];
den = [1, 7, 15, 9, 0];
[r, p] = residue[num, den]
```

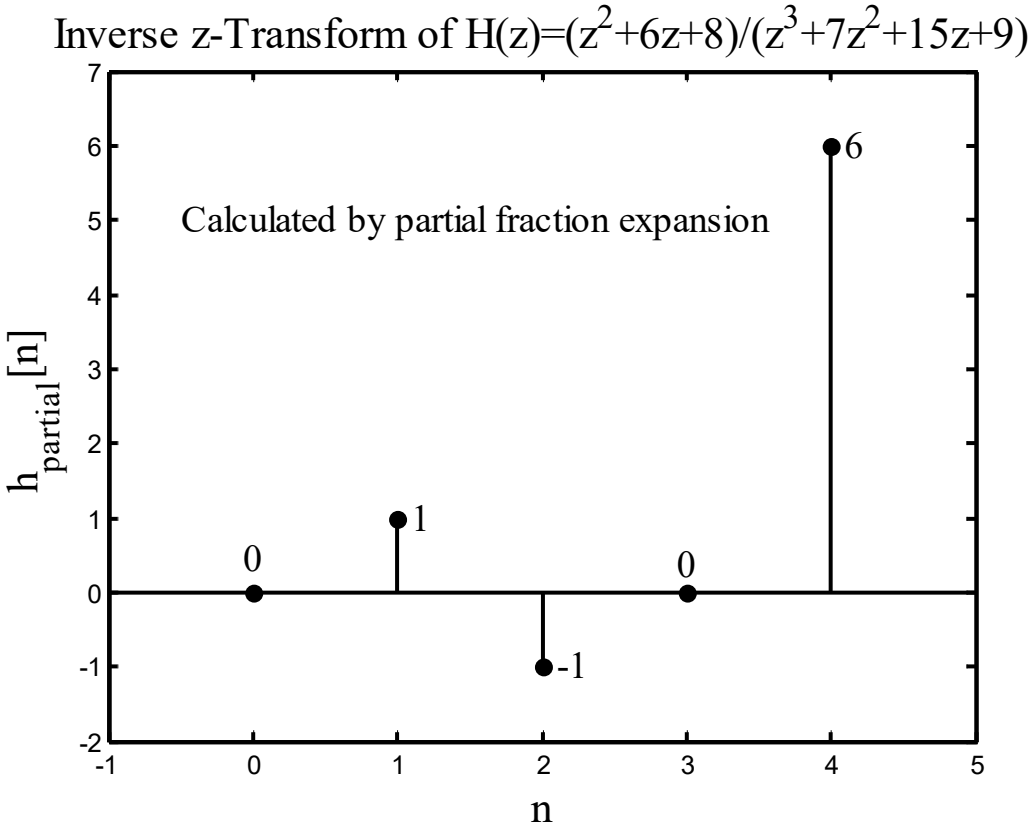
$r = -0.1389 = c_1$	$p = -3$		← Residue of lowest power of pole
$-0.1667 = c_2$	-3		← Residue of highest power of pole
$-0.75 = c_3$	-1		
$0.889 = c_0$	0		

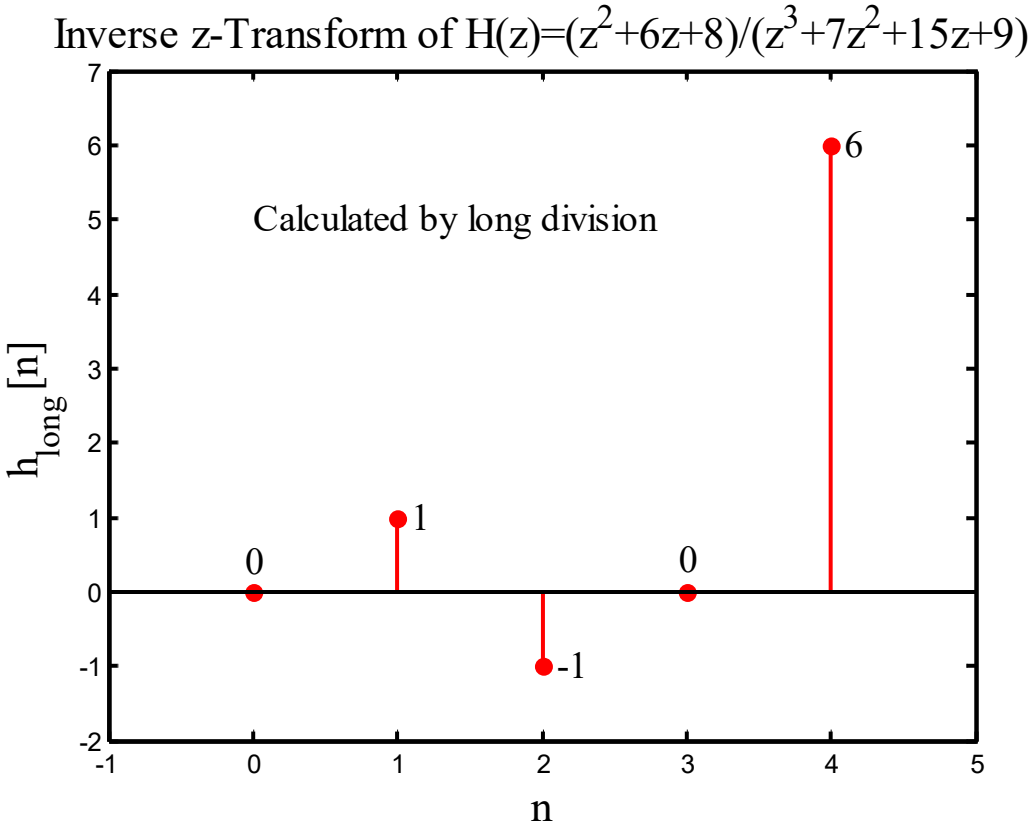
$$H(z) = 8/9 + \frac{(-5/36)z}{z+3} + \frac{(-1/6)z}{(z+3)^2} + \frac{(-0.75)z}{z+1}$$

$$h[n] = 8/9\,\delta[n] + (-5/36)(-3)^n u[n] + (-1/6)\,n(-3)^{n-1}u[n] + (-0.75)(-1)^n u[n]$$

↑ NOT STABLE!

```
% chap7_inverse_z_transform_partial_fractions.m
% Chapter 7 Inverse z-Transform example using partial
% fractions expansion of
%   H(z)=(z^2+6z+8)/(z^3+7z^2+15z+9)
clc;clear;close all;
num = [1,6,8];      % Input coefficients of numerator polynomial
den = [1,7,15,9];   % Input coefficients of denominator polynomial
h = dimpulse(num,den,5);  % Calculate first 5 values of h[n]
n=0:4;
for l=1:length(h),
    k=l-1;
    if(k==0),
        h_exact(l)=8/9-5/36*(-3)^k-1/6*k*(-3)^(k-1)-0.75*(-1)^k;
    else
        h_exact(l)=-5/36*(-3)^k-1/6*k*(-3)^(k-1)-0.75*(-1)^k;
    end
end
stem(n,h,'r.'), axis([-1 5 -2 7]),
ylabel('h_{long }[n]','fontsize',16,'fontname','times'),
xlabel('n','fontsize',16,'fontname','times'),
title('Inverse z-Transform of H(z)=(z^2+6z+8)/(z^3+7z^2+15z+9)',...
    'fontsize',16,'fontname','times'),
text(0,5,'Calculated by long division',...
    'fontsize',16,'fontname','times')
for m=1:length(h),
    text(n(m)+0.1,h(m),['',num2str(h(m),2)],'horizontalalignment',...
        'left','verticalalignment','middle')
end
figure, stem(n,h_exact,'k.'), axis([-1 5 -2 7]),
ylabel('h_{partial }[n]','fontsize',16,'fontname','times'),
xlabel('n','fontsize',16,'fontname','times'),
title('Inverse z-Transform of H(z)=(z^2+6z+8)/(z^3+7z^2+15z+9)',...
    'fontsize',16,'fontname','times'),
text(-0.5,5,'Calculated by partial fraction expansion',...
    'fontsize',16,'fontname','times')
for m=1:length(h),
    text(n(m)+0.1,h_exact(m),['',num2str(h_exact(m),2)],...
        'horizontalalignment','left','verticalalignment','middle')
end
set(findobj('type','line'),'linewidth',1.5)
set(findobj('type','line'),'markersize',18)
set(findobj('type','axes'),'linewidth',2)
set(findobj('type','text'),'fontsize',14,'fontname','times')
```


Inverse z-Transform of H(z)=(z^2+6z+8)/(z^3+7z^2+15z+9)
Calculated by partial fraction expansion
h_partial[n]
n
0
1
0
-1
6


Inverse z-Transform of H(z)=(z^2+6z+8)/(z^3+7z^2+15z+9)
Calculated by long division
h_long[n]
n
0
1
0
-1
6