ex. $$H(z) = \frac{z^2+2z+1}{z^2-z+1}$$ ← Already found the initial values by long division

$$\frac{H(z)}{z} = \frac{z^2+2z+1}{z(z^2-z+1)}$$ ← already found roots

$$= \frac{z^2+2z+1}{z[z-(0.5+j0.866)][z-(0.5-j0.866)]}$$

$$= \frac{C_0}{z} + \frac{C_1}{z-(0.5+j0.866)} + \frac{C_2}{z-(0.5-j0.866)}$$

<u>Find residues/coeff.</u>

$$C_0 = \left[z\,\frac{H(z)}{z}\right]\Big|_{z=0} = \frac{0^2+2(0)+1}{0^2-0+1} = \underline{1}$$

$$C_1 = \left[(z-(0.5+j0.866))\frac{H(z)}{z}\right]\Big|_{z=0.5+j0.866} = \frac{(0.5+j0.866)^2+2(0.5+j0.866)+1}{(0.5+j0.866)[(0.5+j0.866)-(0.5-j0.866)]}$$

$$C_1 = -j1.7321 = \underline{-j\sqrt{3}}$$

$$C_2 = C_1^* = \underline{+j\sqrt{3}}$$

or

$$C_2 = \left[(z-(0.5-j0.866))\frac{H(z)}{z}\right]\Big|_{z=0.5-j0.866}$$ = same as above

ex. cont.

Next, fill-in C_0, C_1, + C_2 and multiply both sides by z to get

$$H(z) = 1 + \frac{(-j\sqrt{3})z}{z-(0.5+j0.866)} + \frac{(j\sqrt{3})z}{z-(0.5-j0.866)}$$

$\downarrow$ using Table 7.3

$$\underline{h[n] = \delta[n] + \underbrace{(-j\sqrt{3})(0.5+j0.866)^n u[n] + (j\sqrt{3})(0.5-j0.866)^n u[n]}_{\text{combine}}}$$

For (7.68), $2|C_1|\sigma^n \cos(\Omega n + \measuredangle C_1)$

$|P_1| = \sigma = |0.5+j0.866| = 1$ $\qquad |C_1| = \sqrt{3}$

$\measuredangle P_1 = \Omega = \measuredangle(0.5+j0.866) = 60° = \pi/3$ $\qquad \measuredangle C_1 = -90° = -\pi/2$

$$h[n] = \delta[n] + 2\sqrt{3}\,(1)^n \cos(\pi/3\, n - \pi/2)\, u[n]$$

$$\underline{h[n] = \delta[n] + 2\sqrt{3}\cos(\pi/3\, n - \pi/2)\, u[n]}$$

Check

$$\left.\begin{aligned}
h[0] &= 1 + 2\sqrt{3}\cos(-\pi/2) = 1 \\
h[1] &= 2\sqrt{3}\cos(\pi/3 - \pi/2) = 3 \\
h[2] &= 2\sqrt{3}\cos(2\pi/3 - \pi/2) = 3 \\
h[3] &= 2\sqrt{3}\cos(3\pi/3 - \pi/2) = 0 \\
h[4] &= 2\sqrt{3}\cos(4\pi/3 - \pi/2) = -3
\end{aligned}\right\} \text{Same as before!}$$

...