Example- 2-point Moving Average filter.

Moving Average (2-point)

$$y[n] = \tfrac{1}{2}\left(x[n] + x[n-1]\right)$$

letting $x[n] = \delta[n]$

$$y[n] = h[n] = \tfrac{1}{2}\left[\delta[n] + \delta[n-1]\right]$$ ← pretty short!

Calculate DTFT of $h[n]$ to get

$$H(\Omega) = \tfrac{1}{2}\left[1 + e^{-j\Omega}\right]$$

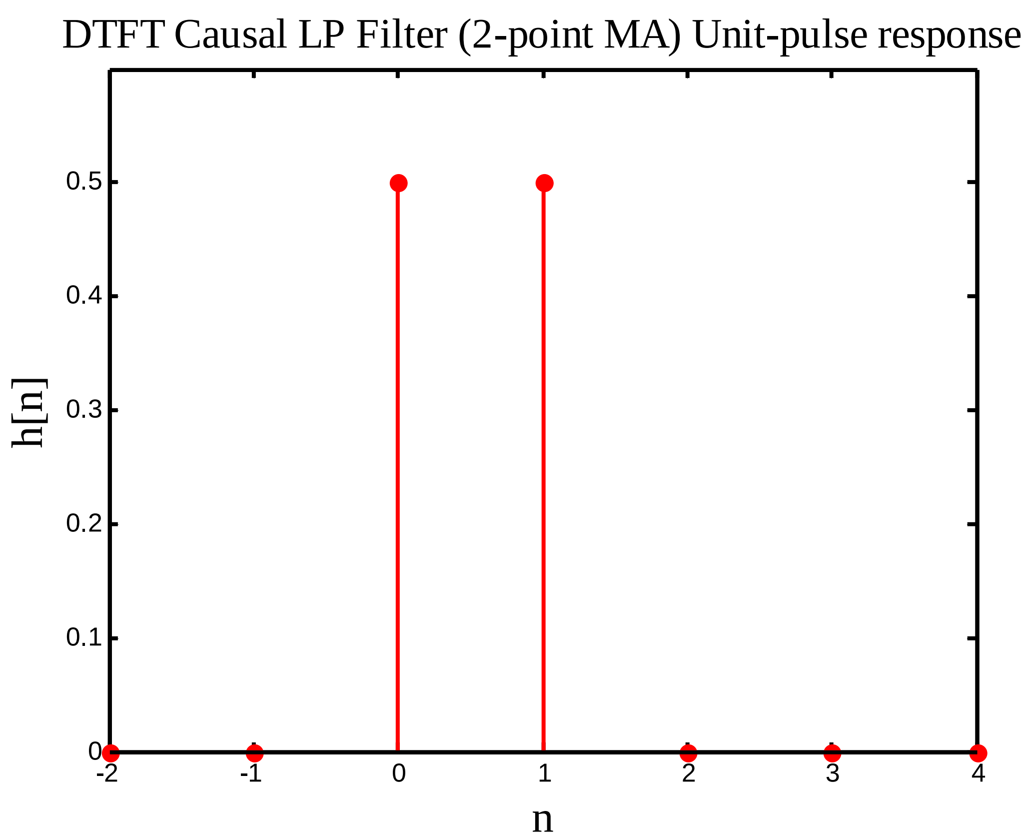
$$= \tfrac{1}{2} e^{-j\Omega/2}\left[e^{+j\Omega/2} + e^{-j\Omega/2}\right]$$

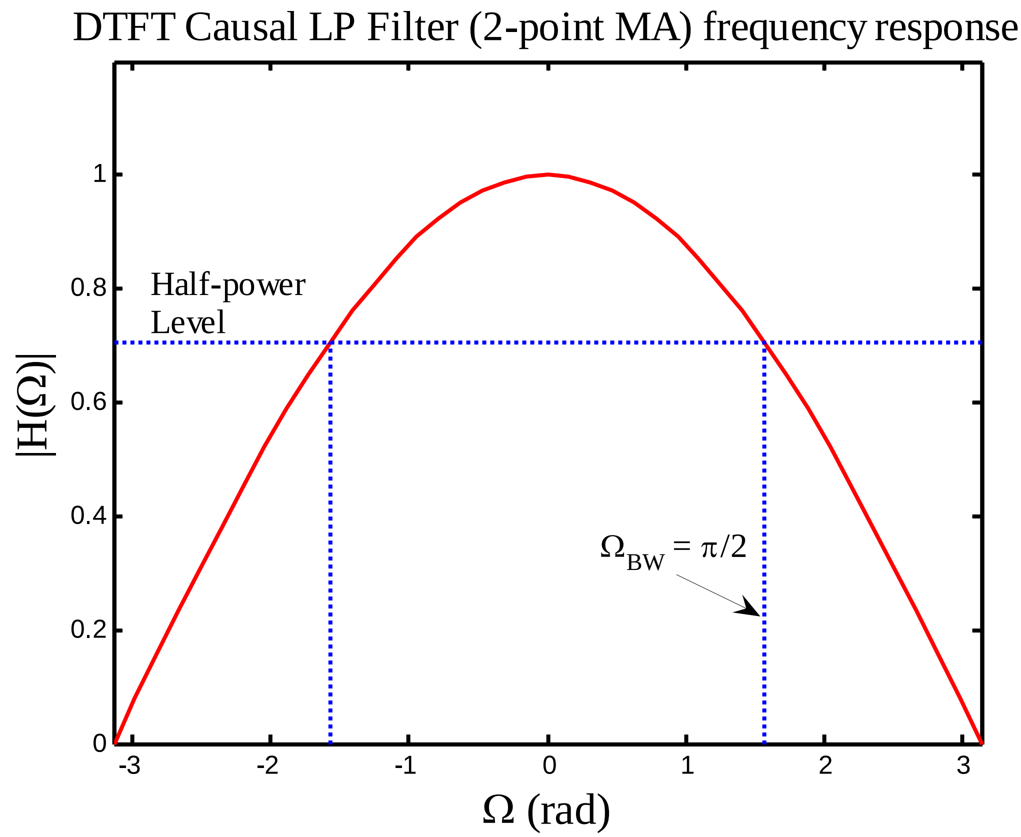
$$= \tfrac{1}{2} e^{-j\Omega/2}\left[2\cos(\Omega/2)\right]$$

$$= \cos(\Omega/2)\, e^{-j\Omega/2}$$ ← implies time-delay of ½ an index step

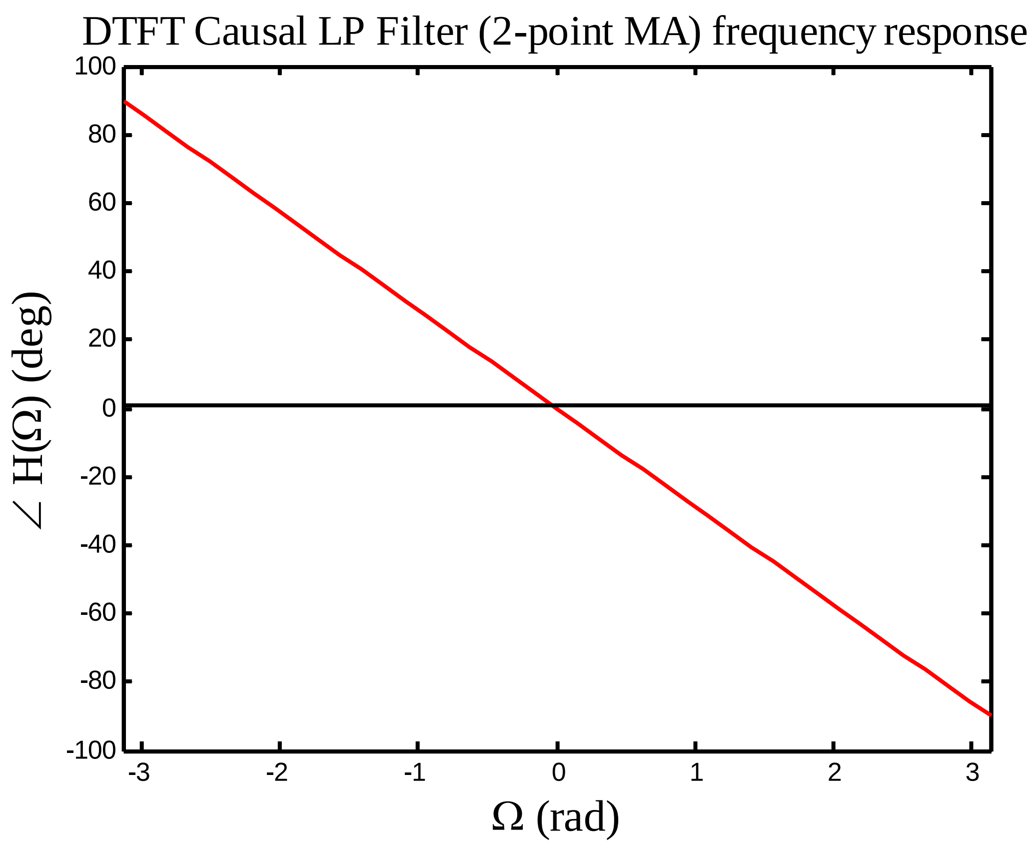
⇒ Show plots

```
% chap5_causal_MA_lowpass_filter.m
% Chapter 5 Causal 2-point moving average (MA)
% lowpass filter with diff. eqn-
%    y[n] = 0.5*(x[n]+x[n-1])
% and corresponding unit-pulse response
%    h[n] = 0.5*{delta[n]+delta[n-1]}.
clear;close all;clc;
n = -2:1:4; h=zeros(1,length(n)); h(3)=0.5; h(4)=0.5;
Omega = -pi:0.05*pi:pi;  H = 0.5+0.5*exp(-j*Omega);
stem(n,h,'r.'),axis([-2 4 0 0.6]),
xlabel('n','fontsize',16,'fontname','times'),
ylabel('h[n]','fontsize',16,'fontname','times'),
title('DTFT Causal LP Filter (2-point MA) Unit-pulse response',...
    'fontsize',15,'fontname','times'),
figure, plot(Omega,abs(H),'r-',[-pi,pi],[0.707,0.707],'b:',...
    [-pi/2,-pi/2],[0,0.707],'b:',[pi/2,pi/2],[0,0.707],'b:'),
axis([-pi pi 0 1.2]),
xlabel('\Omega (rad)','fontsize',16,'fontname','times'),
ylabel('|H(\Omega)|','fontsize',16,'fontname','times'),
title('DTFT Causal LP Filter (2-point MA) frequency response',...
    'fontsize',15,'fontname','times'),
figure, plot(Omega,angle(H)*180/pi,'r',[-pi,pi],[0.707,0.707],'k-'),
axis([-pi pi -100 100]),
ylabel('\angle H(\Omega) (deg)','fontsize',16,'fontname','times'),
xlabel('\Omega (rad)','fontsize',16,'fontname','times'),
title('DTFT Causal LP Filter (2-point MA) frequency response',...
    'fontsize',15,'fontname','times'),
```





DTFT Causal LP Filter (2-point MA) frequency response
Half-power
Level
$\Omega_{BW} = \pi/2$
$|H(\Omega)|$
Ω (rad)
1
0.8
0.6
0.4
0.2
0
-3
-2
-1
0
1
2
3



DTFT Causal LP Filter (2-point MA) frequency response
$\angle H(\Omega)$ (deg)
Ω (rad)
100
80
60
40
20
0
-20
-40
-60
-80
-100
-3
-2
-1
0
1
2
3

Good → linear phase

Not-so-good → slow roll-off (poor lowpass)

* Note that the half-power point $H(\Omega) = 0.707$ occurs @ $\Omega = \pi/2$. ⇒ only frequencies near $\Omega = \pi$ get seriously attenuated.

⇓

* Also, effective BW of filter determined by sampling rate

e.g. $x[n] = A\cos(\Omega_0 n)$ is the input

where $\Omega_0 = \omega_0 T$ is determined by T!

So, if T is small, $\Omega_0 \to 0$ & $H(\Omega) \to 1$.

If T is bigger, $\Omega_0 \to \pi$ & $H(\Omega) \to 0$.