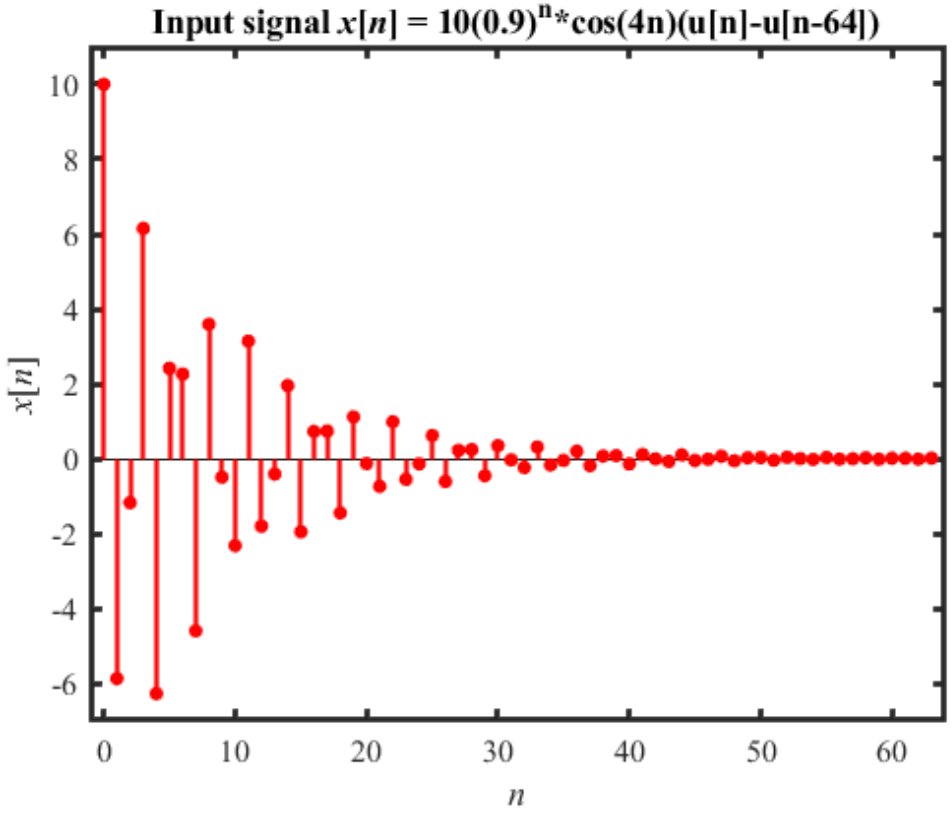
Example- In this example, we will compute the DFT of the discrete-time (DT) function-

$$x[n] = 10(0.9)^n \cos(4n) \text{ for } n = 0, 1, 2, \ldots 63, \text{ or}$$

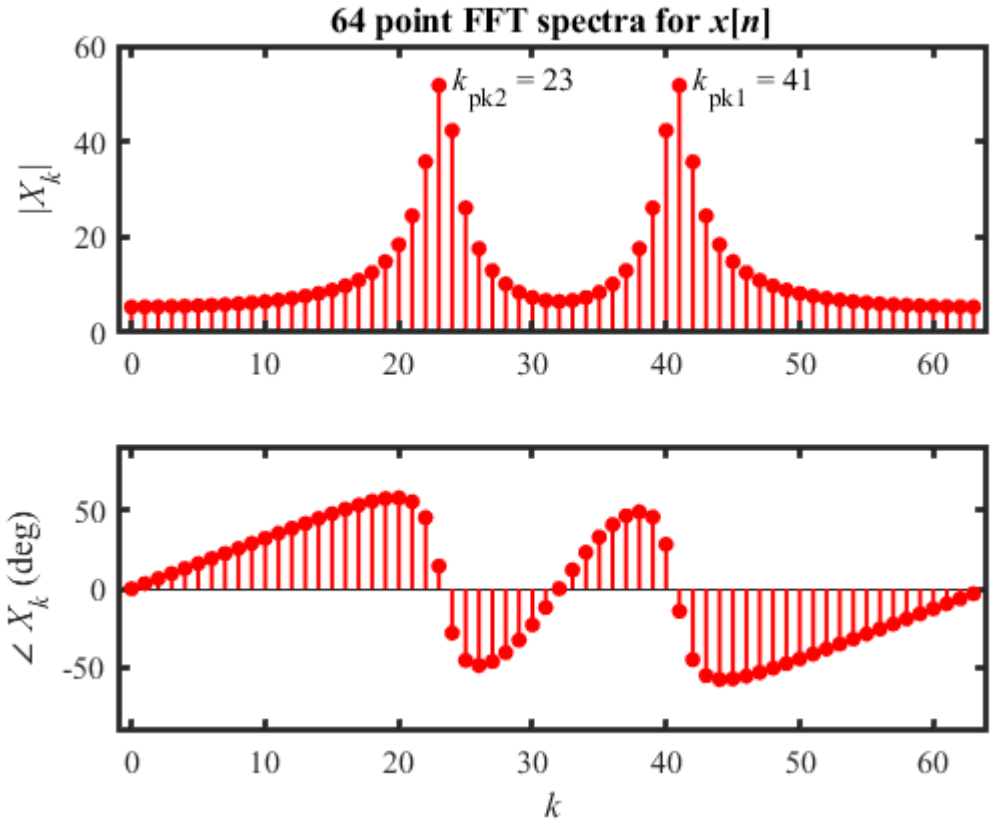
$$x[n] = [10(0.9)^n \cos(4n)][\, u[n] - u[n - 64]].$$

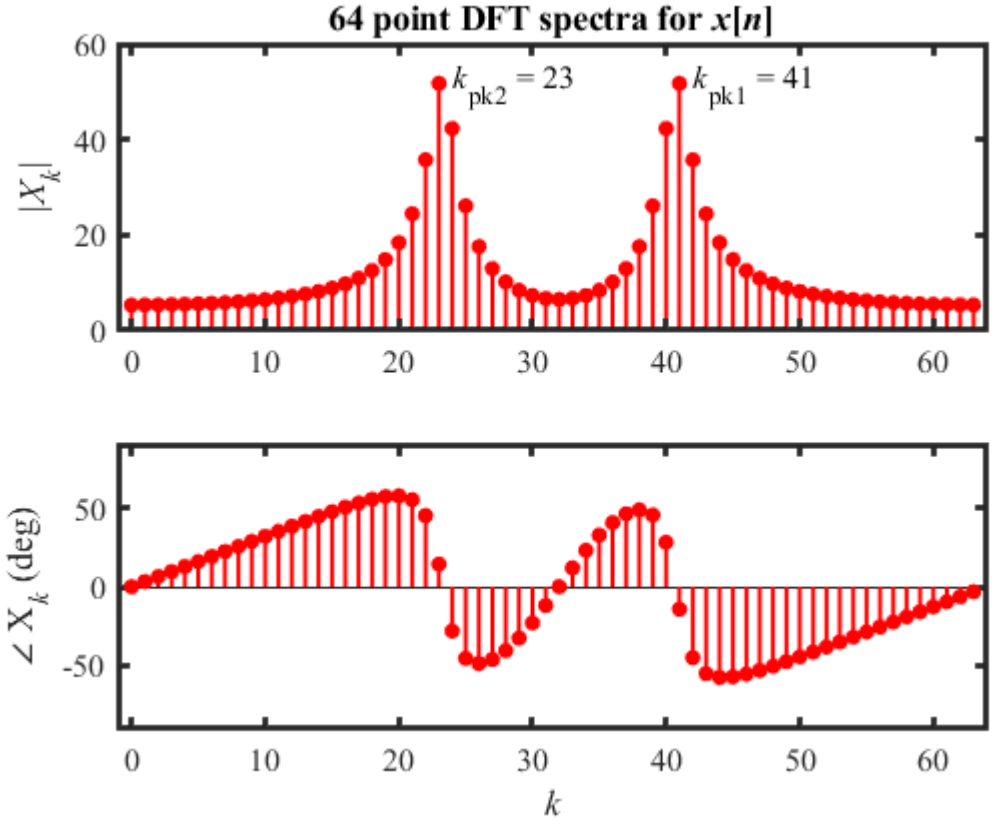
both directly and using the fast Fourier transform (FFT) algorithm. The results should be identical since $N = 64 = 2^6$.



From the $\cos(4n)$ term in $x[n]$, we can expect peaks in DFT/FFT results at indices k corresponding to DT frequency $\Omega = 4$ rad. Using the relation between DFT (e.g., X_k) and DTFT (e.g., $X(\Omega)$) frequencies $\Omega = \frac{2\pi k}{N}$. Substituting in $N = 64$ and $\Omega = 4$ rad and solving for k leads to $k_{\text{pk1}} = \frac{64 \bullet 4}{2\pi} = 40.7 \Rightarrow \underline{k_{\text{pk1}} \simeq 41}$. Given X_k is symmetric about $k = \frac{N}{2} = 32$, we also expect a peak at $k_{\text{pk2}} = \frac{N}{2} - \Delta = \frac{64}{2} - \left(41 - \frac{64}{2}\right) \Rightarrow \underline{k_{\text{pk2}} = 23}$.

```
% Chapter 4 FFT Example 1 (chap_04_fft_example_1.m)
% Compute FFT and DFT of 64 point input
% x[n] = 10(0.9)^n*cos(4n)(u[n]-u[n-64])
clear;clc;close all;
N = 64; n = 0:1:N-1;  % time indices
Omega = 4; kpk1 = round(N*Omega/2/pi);
kpk2 = N/2 - (kpk1 - N/2);
x = 10*(0.9).^n.*cos(Omega*n);  % Define the input
Xkfft = fft(x);  Xkdft = dft(x);   % Call FFT & DFT functions
Xkmagfft=abs(Xkfft); Xkangfft=angle(Xkfft)*180/pi; % FFT line spectra
Xkmagdft=abs(Xkdft); Xkangdft=angle(Xkdft)*180/pi; % DFT line spectra
kfft = 0: 1 : length(Xkfft)-1; % indices for FFT
kdft = 0: 1 : length(Xkdft)-1; % indices for DFT
% Plot input signal
stem(n,x,'r.','linewidth',1.6,'markersize',18), axis([-1 N -7 11]),
ylabel('{\itx}[{\itn}]','fontsize',14,'fontname','times'),
xlabel('{\itn}','fontsize',14,'fontname','times'),
title('Input signal {\itx}[{\itn}] = 10(0.9)^n*cos(4n)(u[n]-u[n-64])',...
    'fontsize',16,'fontname','times'),
figure, % Plot FFT amplitude and phase spectra
subplot(211),stem(kfft,Xkmagfft,'r.','linewidth',1.5,'markersize',20),
axis([-1 N 0 60]),
text(kpk1+1,Xkmagfft(kpk1+1),['{\itk}_{pk1} = ',num2str(kpk1)],...
        'horizontalalignment','left','verticalalignment','middle'),
text(kpk2+1,Xkmagfft(kpk2+1),['{\itk}_{pk2} = ',num2str(kpk2)],...
        'horizontalalignment','left','verticalalignment','middle')
ylabel('|{\itX}_{\itk}|','fontsize',14,'fontname','times'),
title('64 point FFT spectra for {\itx}[{\itn}]',...
    'fontsize',16,'fontname','times'),
subplot(212),stem(kfft,Xkangfft,'r.','linewidth',1.5,'markersize',20),
axis([-1 N -90 90]),
ylabel('\angle {\itX}_{\itk} (deg)','fontsize',14,'fontname','times'),
xlabel('{\itk}','fontsize',14,'fontname','times'),
figure, % Plot DFT amplitude and phase spectra
subplot(211),stem(kdft,Xkmagdft,'r.','linewidth',1.5,'markersize',20),
axis([-1 N 0 60]),
text(kpk1+1,Xkmagdft(kpk1+1),['{\itk}_{pk1} = ',num2str(kpk1)],...
        'horizontalalignment','left','verticalalignment','middle')
text(kpk2+1,Xkmagdft(kpk2+1),['{\itk}_{pk2} = ',num2str(kpk2)],...
        'horizontalalignment','left','verticalalignment','middle')
ylabel('|{\itX}_{\itk}|','fontsize',14,'fontname','times'),
title('64 point DFT spectra for {\itx}[{\itn}]',...
    'fontsize',16,'fontname','times'),
subplot(212),stem(kdft,Xkangdft,'r.','linewidth',1.5,'markersize',20),
axis([-1 N -90 90]),
ylabel('\angle X_{\itk} (deg)','fontsize',14,'fontname','times'),
xlabel('{\itk}','fontsize',14,'fontname','times'),
set(findobj('type','line'),'linewidth',1.5,'markersize',14)
set(findobj('type','axes'),'linewidth',2,'fontsize',12,'fontname','times')
set(findobj('type','text'),'fontsize',12,'fontname','times')
```





<u>Same results</u>! Note spikes/peaks in $|X_k|$ indeed occur at k = 23 & 41.